ASIACOMB 2026

Program Book

Complete Online Edition

Daejeon, Republic of Korea

August 24–28, 2026

Contents

Monday, August 24

Sunday, August 23: Registration | 15:00–18:00 | DCC Exhibition Hall 1, first floor

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08:30–09:15	**Registration** | DCC Exhibition Hall 1, first floor					
09:15–09:30	**Opening Remarks** | DCC 2nd floor					
09:30–10:30	**Plenary Talk — Mihyun Kang** | Phase transitions in random graphs on surfaces | DCC 2nd floor					
10:30–11:00	**Coffee Break**					
11:00–11:30	**Yi-Hau Kao** Proving Guo's Conjecture on Richardson Tableaux	**Yijia Fang** Canonical Ramsey Property for All Triangles	**Nikolai Karol** Structure of k-Matching-Planar Graphs	**Myungho Choi** 2-limited broadcast domination in cubic graphs	**Tomoki Tamaru** More on the Corner-Vector Construction for Spherical Designs	**Jaehyeon Seo** Degree-sequence bounds for independent sets via multivariate local occupancy
11:30–12:00	**Mike Cummings** Webs and Smooth Components of Two Column Springer Fibers	**Matías Azócar Carvajal** Canonical Ramsey theorem for graphs with clean intersections	**Rongxing Xu** Partitioning triangle-free planar graphs into a forest and a linear forest	**Sonwabile Mafunda** On the conjecture of Beineke and Henning on independent distance dominating sets	**Yifan Jing** Sharp quantitative stability in Hilbert spaces	**Fan Chang** Functional Inequalities and Random Walks on Increasing Subsets of the Hypercube
12:00–14:30	**Lunch Break**					
14:30–15:30	**Plenary Talk — Anton Bernshteyn** | Matchings and the Banach–Tarski Paradox | DCC 2nd floor					
15:30–16:00	**Coffee Break**					
16:00–16:30	**Masato Kobayashi** A quite simple formula on Bruhat order and bigrassmannian permutations	**Lin-Peng Zhang** Perfect tilings with the generalised triangle in k-graphs	**Yoshimi Egawa** Generation theorem for 5-connected graphs	**Yakov Shubin** On supersaturation in the Erdős-Sós Problem	**Arsenii Sagdeev** Graphs in Euclidean Ramsey Theory	**Ningyuan Yang** Chromatic thresholds for linear equations and recurrence
16:30–17:00	**Eunjeong Lee** Intersections of Schubert varieties and smooth T-stable subvarieties of flag varieties	**Nicolás Sanhueza Matamala** Optimal and efficient partite decompositions of hypergraphs	**Lili Hao** The existence of 2-factors in $3/2$-tough maximal planar graphs	**Ligang Jin** Coloring of generalized signed graphs and its application to DP coloring	**Laurentiu Ploscaru** Many Antipodal Pairs Force many Neighboring Pairs	**Semin Yoo** Multiplicative irreducibility of shifted multiplicative subgroups
17:00–17:30	**Hyunwoo Lee** Anticoncentration of random spanning trees in almost regular graphs	**Arjun Ranganathan** Exact minimum co-degree bounds for tight Hamilton cycles	**Xiang Chen** A constructive characterization of uniformly 4-connected graphs	**Xiaowei Yu** (Fractional) balanced coloring of signed graphs	**Maxim Klimenko** Covering integer points of a cross-polytope by subspaces	**Guo-Dong Hong** Simultaneous popular polynomial Szemerédi theorem over finite fields
18:00 onward	**Reception** | IBS Science Culture Center, 3rd floor					

Tuesday, August 25

Time	Room A	Room B	Room C	Room D	Room E	Room F
09:00–10:00	**Plenary Talk — Van Vu** | New Matrix Perturbation Bounds via Relative Norm | DCC 2nd floor					
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10:30–11:00	**Yulin Peng** Coxeter Condorcet domains and Condorcet root posets	**Jing Wang** Spectral Turán Problems for Expansion Hypergraphs	**Alexander Clow** Small Quasi-Kernel in Digraphs with Bounded Out-Degree	**Chenglong Deng** List colouring of toroidal grids	**Jie Wang** Lower bound theorems on the numbers of faces of polytopes with at most $3d-1$ vertices	**Alexander Gavrilyuk** Intersecting families of spanning trees in complete bipartite graphs
11:00–11:30	**Hanlin Xu** MacNeille completions of parabolic quotients in the symmetric group	**Zion Hefty** Improving $R(3,k)$ in just two bites	**Seokbeom Kim** The structure of $\Delta(1,2,2)$-free tournaments	**Zihui Xu** Between proper and square colorings of planar graphs with maximum degree at most four	**David Yost** Decomposability of polyhedra and their graphs	**Hideki Matsumura** Ellipsoidal designs and the Prouhet–Tarry–Escott problem
11:30–12:00	–	**Guanghui Wang** Erdős-Rogers function on hypergraphs	**Bartlomiej Kielak** Digraphs with density maximized by transitive tournaments	**Xiaolan Hu** Planar graphs of odd girth 7 are fractional $14/5$-colorable	**Mangaldeep Saha** Normal 4-pseudomanifolds with a relative 2-skeleton	**Yetong Sha** Spanning trees of bounded degree in random geometric graphs
12:00–14:30	**Lunch Break**					
14:30–15:30	**Plenary Talk — Joshua Zahl** | (Discretized) incidence geometry, harmonic analysis, and geometric measure theory | DCC 2nd floor					
15:30–16:00	**Coffee Break**					
16:00–16:30	**Kai Zhang** The lattices $m\times 2$ and $m\times 3$ are not Schur positive	**Dylan King** Relative Ordered Turán Densities	**Hikaru Yokoi** Self-duality of pathwidth for polyhedral embeddings	**Cyril Pujol** Winding number and circular coloring	**Ferdinand Ihringer** Local arcs, locally repairable codes, large matchings	**Eduard Inozemtsev** Frankl's diversity theorem for permutations
16:30–17:00	**Hojoon Lee** Positivity Phenomena in Koornwinder Moments for the 2-ASEP	**Henry Liu** On degree powers in the degenerate Turán problem	**On-Hei Solomon Lo** A characterization of graphs with no $K_{3,4}$-minor	**Yiting Jiang** Indicated List Coloring Game On Graphs	**Ananth Ravi** The chromatic number of finite projective spaces	**Elizaveta Iarovikova** A complete t-intersection theorem for families of spanning trees
17:00–17:30	–	**Fan Yang** Extremal density for subdivisions with length or sparsity constraints	**Shinya Fujita** From Halin's Edge Removability to Matching Removability in k-connected graphs	**Naoki Matsumoto** Cubic graphs with game chromatic number 3	**Lukas Klawuhn** Designs of Perfect Matchings	**Andrey Kupavskii** Intersection theorems in spread domains

Poster Presentations | Tuesday through Friday morning on the 2nd floor of DCC

Yen-Jen Cheng — On q-counting of non-crossing chain and parking functions
Takumi Iwasaki — Forbidden subgraphs and the existence of covers and partitions of graphs by paths and cycles
Yasuko Matsui — Enumeration of Minimum-Cost Compact Edge-Colorings of Trees
Masaya Tomie — Bipancyclicity of the generalized Fibonacci cube $B_n(110)$
Soura Sena Das — On oriented total coloring of sparse graphs
Márton Marits — Covering graphs by subgraphs with bounded fractional chromatic number
Norihiro Nakashima — Weight Enumerators as Quasi-Polynomials and the Periodicity of Minimum Weights
Wenbin Wang — Non-bipartite extremal number for 3-chromatic graphs with a critical-edge

Wednesday, August 26

Time	Room A	Room B	Room C	Room D	Room E	Room F
09:00–10:00	**Plenary Talk — Alexander Postnikov** | Honeycombs, plabic graphs, and polypositroids | DCC 2nd floor					
10:00–10:10	**Group Photo**					
10:10–10:30	**Coffee Break**					
10:30–11:00	**Xinbei Liu** Lascoux's series, parking functions and noncrossing partitions	**Zhifei Yan** Monochromatic matchings in hypergraphs	**Taehee Hong** p-competitively orientable graphs	**Eckhard Steffen** Edge-coloring 4- and 5-regular projective planar graphs with no Petersen-minor	**Haofang Sun** Curvilinear Tilings and Hurwitz Problem	**Yeonsu Chang** Algorithmic Applications of Reduced Component Max-Leaf
11:00–11:30	**Michael Ruofan Zeng** The K_0-Ring of Spanning Line Configurations, Fubini Words, and Rectangular Pipe Dreams	**Minghui Yu** Berge tight cycles of all lengths in hypergraphs	**Chaoliang Tang** An Improved Bound of Stein's Conjecture for Antidirected Paths	**Jialu Zhu** Degree-truncated choice number of planar graphs	**Shohei Koizumi** Spanning triangulations in plane X-mosaics	**Reymond Akpanya** Regular Graphs with given Automorphism Groups
11:30–12:00	**Minho Song** Deograms and their links to rational Catalan objects	**Yulin Yang** An Erdős matching conjecture for vector spaces	**Alexander Clifton** Locally Irregular Graph Covering	**Kenta Ozeki** Odd edge colorings of graphs with high connectivity	**Dohyeon Lee** Helly-type Theorems for Multiple Piercing	**Colin Geniet** Basis Number of Graphs Excluding Minors
12:00–14:30	**Lunch Break**					
14:30–15:30	**Plenary Talk — Jaehoon Kim** | Local Conditions for Global Structures in Graphs | DCC 2nd floor					
15:30–15:40	**Break**					
15:40–16:10	**Prize Ceremony** | DCC 2nd floor					
16:10–17:10	**Prize Winner Talk** | Name: TBA / Title: TBA / Location: DCC 2nd floor					
17:10–18:00	**Break**					
18:00 onward	**Banquet** | DCC 2nd floor					

Thursday, August 27

Time	Room A	Room B	Room C	Room D	Room E	Room F
09:00–10:00	**Plenary Talk — Allen Knutson** | Periodic pipe dreams and matrix positroid varieties | DCC 2nd floor					
10:00–10:30	**Coffee Break**					
10:30–11:00	**Sen-Peng Eu** On Ternary Trees and Fighting Fish	**Chenyang Zhang** Sharp bounds for uniform union-free hypergraphs	**Masahiro Sanka** Hamiltonicity of $P_2 \cup kP_1$-free graphs with many vertices	**Xujun Liu** Between proper and square colorings of sparse graphs	**Shohei Satake** On spectrally indistinguishable pseudorandom graphs	–
11:00–11:30	**Shen-Fu Tsai** On pattern-avoiding hypermatrices and higher-dimensional partitions	**Dilong Yang** Relative discrepancy of hypergraphs	**Leilei Zhang** Hamiltonian Properties of 3-Connected Claw-Free Graphs and Line Graphs of 3-Hypergraphs	**Qiancheng Ouyang** New bounds for proper h-conflict-free colourings	**Wanting Sun** Clique factors in random samplings of regular graphs	**Toby Insley** Sparse Partitions of Graphs with Bounded Clique Number
11:30–12:00	**Tamás Takács** Prefix-bounded matrices	**Jing Yu** Hypergraph independence bounds: from maximum degree to average degree	**Kengo Enami** Induced outerplanar subgraphs in planar graphs	**Yumiko Ohno** The achromatic number and the pseudoachromatic number of caterpillars	**Yaobin Chen** Note on the trace of random walks on pseudorandom graphs	**Pawel Pralat** Top Trading Cycles with Random Preferences
12:00–14:30	**Lunch Break**					
14:30–15:30	**Plenary Talk — Bojan Mohar** | Back to the basics — The Four-Color Theorem and its generalizations | DCC 2nd floor					
15:30–16:00	**Coffee Break**					
16:00–16:30	**Péter Pál Pach** On the density of Kravitz sets	**Wenling Zhou** Some exact values of the uniform Turán densities	**Mujin Choi** Odd-Cycle-Packing-treewidth and Grid Theorem for odd-minor relation	**Yian Xu** Polynomial chi-boundedness of graphs forbidding specific forests	**Alexander Natalchenko** Anti-Ramsey Numbers of Expansions of Graphs	**Zhenyu Li** A step toward Chen-Lih-Wu conjecture
16:30–17:00	**Jiali Du** On m-partite (di)graphical semiregular representation of finite groups	**Yuefang Sun** On oriented Turán problems	**Jungho Ahn** Unavoidable pivot-minors in graphs of large rank-depth	**Yisai Xue** On the chromatic profile for tripartite graphs and beyond	**Miklos Ruszinko** Monochromatic paths and components	**Santhosh Raghul** θ-free graphs: characterization and consequences
17:00–17:30	**Jaeseong Oh** Shuffle theorem for torus link homology	**Laihao Ding** Vanishing orders and zero degree Turán densities	**Jacob Stegemann** A duality theorem for infinite tree-width	**Yangyan Gu** Online list version of Hadwiger's conjecture	**Georgy Sokolov** On the Erdős-Kleitman problem	**Ryota Matsubara** On bipartite holes and k-leaf-connectedness

Friday, August 28

Time	Room A	Room B	Room C	Room D	Room E	Room F
09:00–10:00	**Plenary Talk — Jie Han** | Random sparsification of graphs | DCC 2nd floor					
10:00–10:30	**Coffee Break**					
10:30–11:00	**Mikhail Bludov** On the Homotopy Type of Unbalanced Subset Complexes	**Guorong Gao** Almost regular subgraphs under spectral radius constraints	**Masaki Kashima** New conditions ensuring specified factors of graphs	**Jung Hon Yip** Hadwiger's Conjecture for $\{co-claw, co-gem\}$-free graphs and $\{fork, antifork\}$-free graphs	**Gang Yang** Boolean lattice without small rainbow subposets	**Shiqi Cao** Dowling's polynomial conjecture for independent sets of matroids
11:00–11:30	**Daniel McGinnis** Multi-extremal Betti numbers	**Jialin He** On the Number of Triangles in K_4-Free Graphs	**Koshin Yoshida** Partitions of a bipartite graph into cycles containing specified paths	**Jan Ouborny** All graphs are majority 3-choosable	–	–
11:30–12:00	**Maryam Mohammadi Yekta** A lower bound for the coefficients of denormalized Lorentzian Laurent series and applications	**Donglei Yang** Packing 4-cliques in edge-weighted graphs	**Takahiro Ueoro** On graphs without cycles of length $0 \bmod 3$ or $4 \bmod 6$	**Luis Kuffner** Homomorphism bounds for $(K_4, -)$-minor-free signed graphs of negative girth k	**Nikolai Terekhov** On the problem of Füredi concerning (k, L)-systems	**Yoshio Sano** On matching preclusion sets in weighted graphs
12:00–14:30	**Lunch Break**					
14:30–15:00	**Donghyun Kim** Exploring the science fiction	**Dingyuan Liu** Turán problems for simplicial complexes	**Ravindra Pawar** Matching Minors: a sequel to the results of Lovász and Plummer	**O-joung Kwon** Induced-packing variants of the Erdős-Posa theorem	**Ingyu Baek** Improved bounds for loose odd cycle densities	**Shunichi Maezawa** On Reconfiguration Graphs Induced by Rainbow Spanning Trees
15:00–15:30	**Warut Thawinrak** Counting lattice points in generalized permutohedra from A to B	**Ni Luh Dewi Sintiari** A Positive Instance of Scott's Conjecture on Induced Subdivisions	**Márk Hunor Juhász** Matching Problems in Temporal Graphs	**Sarah Houdaigoui** A quasi-polynomial bound for the minimal excluded minors for a surface	**Xinqi Huang** Accumulation Points of Homomorphism Thresholds	**Florian Lehner** Three Cops Win on the Torus (even if only two can move at a time)

Plenary Talk Abstracts

Plenary Talk

Phase transitions in random graphs on surfaces

Mihyun Kang (Graz University of Technology)

Monday, August 24 | 09:30–10:30

In light of classic results on Erdős–Rényi random graphs, we discuss recent results on phase transitions in random graphs embeddable on orientable surfaces of constant genus. The main focus is on how imposing the genus constraint affects the global and local structure of random graphs, such as their component structure, local limits, and maximum degree.

Plenary Talk

Matchings and the Banach–Tarski Paradox

Anton Bernshteyn (University of California, Los Angeles)

Monday, August 24 | 14:30–15:30

The Banach–Tarski Paradox is the notoriously counterintuitive fact that a 3-dimensional ball can be transformed into two copies of itself by cutting it into finitely many pieces and rearranging them. The usual explanation for why this could be possible even in principle is that the proof uses the Axiom of Choice and is therefore inherently non-constructive. Contrary to this view, this talk will be about constructive approaches to Banach–Tarski. The right framework to think about such problems turns out to be graph-theoretic, namely using matchings in locally finite bipartite graphs. I will present some new general results in this setting. Based on joint work with Matt Bowen and Felix Weilacher.

Plenary Talk

New Matrix Perturbation Bounds via Relative Norm

Van Vu (University of Hong Kong)

Tuesday, August 25 | 09:00–10:00

Matrix-perturbation bounds quantify how the spectral characteristics of a baseline matrix A change under additive noise E.

Classical results, including Weyl's inequality for eigenvalues and the Davis–Kahan theorem for eigenvectors and eigenspaces, have long played a foundational role in mathematics, and widely used in combinatorics and random matrix theory. These bounds are known to be sharp in worst-case analysis.

In the last 10 years, we have been working to develop a perturbation framework that leverages the interaction between E and the eigenvectors of A, leading to the notion of relative norm, which can be used to replace the operator norm of E in many applications. This perspective yields quantitative improvements over classical bounds, particularly when E is random, a common scenario in applications. In this case, one typically obtains a saving of order square root n, where n is the dimension.

This talk surveys these developments and main ideas, focusing on recent results concerning eigenspace perturbations and low rank approximation. We will discuss applications in random graph theory and estimating covariance matrices.

(joint work with Phuc Tran, Vin University)

Plenary Talk

(Discretized) incidence geometry, harmonic analysis, and geometric measure theory

Joshua Zahl (Nankai University)

Tuesday, August 25 | 14:30–15:30

Incidence geometry is a branch of extremal combinatorics that studies questions of the following form: What is the maximum possible number of incident point–line pairs that can be created by an arrangement of n points and n lines in the plane, or more generally by n points and n curves? What is the maximum number of tangencies that can be created by an arrangement of n circles, or by other types of curves?

We can discretize these problems by selecting a small positive resolution scale δ, and asking analogous questions where we cannot distinguish between objects that are genuinely incident and those that are δ-close to being incident. I will discuss several results in incidence geometry, their discretized counterparts, and applications of these results to problems in harmonic analysis and geometric measure theory.

Plenary Talk

Honeycombs, plabic graphs, and polypositroids

Alexander Postnikov (Massachusetts Institute of Technology)

Wednesday, August 26 | 09:00–10:00

The celebrated work of Knutson and Tao used honeycombs to solve Horn's conjecture on eigenvalues of sums of Hermitian matrices. Honeycombs are simple geometric/combinatorial objects that are linked to many areas of mathematics. We will discuss the evolution of concepts: Littlewood–Richardson coefficients, Berenstein–Zelevinsky patterns, honeycombs, plabic graphs, totally positive Grassmannian, polypositroids. We will also discuss an axiomatic approach to honeycombs and their generalizations. The two core properties of these objects are "convexity" and "duality". The talk is based in part on a joint work with Ankur Moitra and Dora Woodruff.

Plenary Talk

Local Conditions for Global Structures in Graphs

Jaehoon Kim (Korea Advanced Institute of Science and Technology)

Wednesday, August 26 | 14:30–15:30

What local conditions ensure that a network contains a desired global structure? A classical result by Dirac (1952) shows that any n-vertex graph with the minimum degree at least $n/2$ contains a Hamilton cycle, sparking decades of research into Dirac-type theorems. These theorems identify simple local conditions that guarantee the existence of spanning subgraphs—such as cycles, trees, clique factors, and more.

In this talk, I will highlight both classical milestones and several of my own contributions to this theory. These include new Dirac-type results for pseudorandom graphs, advances in smoothed analysis, results on edge-colored graphs, and edge-decomposition theorems. This talk is aimed at a broad audience and will emphasize conceptual insights over technical detail.

Plenary Talk

Periodic pipe dreams and matrix positroid varieties

Allen Knutson (Cornell University)

Thursday, August 27 | 09:00–10:00

Schubert polynomials were introduced in 1982 to (nonuniquely) represent Schubert classes in flag varieties. The polynomials themselves were shown to be the (equivariant) fundamental classes of matrix Schubert varieties, and the classic pipe dream formula for them also was given geometric meaning, in 2005. A cylindrical version of pipe dreams was introduced recently by [Fan–Guo–Su–Xiong] to (nonuniquely) give representatives for the Chern–Schwartz–MacPherson classes of positroid varieties in Grassmannians.

We introduce matrix positroid varieties, and show geometrically that their CSM classes match this [FGSX] formula on the nose. The definition of these varieties involves a surprising twist by a generic diagonal matrix, not visible at the Grassmannian level. This work is joint with Paul Zinn-Justin.

Plenary Talk

Back to the basics — The Four-Color Theorem and its generalizations

Bojan Mohar (Simon Fraser University and University of Ljubljana)

Thursday, August 27 | 14:30–15:30

The Four-Color Theorem had profound influence on the developments of graph theory. It was proved 50 years ago by Appel and Haken. However, its proof left many questions unanswered. Is this result just a coincidence, or are there deeper reasons behind it?

For several years, we try to provide deeper understanding why the 4CT is true. The talk will give an overview of recent breakthrough results obtained in collaboration with Ken-ichi Kawarabayashi, Carsten Thomassen, Mikkel Thorup, Yuta Inoue, Atsuyuki Miyashita, and Tomohiro Sonobe.

Plenary Talk

Random sparsification of graphs

Jie Han (Beijing Institute of Technology)

Friday, August 28 | 09:00–10:00

There has been a recent focus on the properties of the p-random subgraph of a given graph, where each edge is kept independently with probability p. We will introduce some recent advances on these problems, including our results on perfect matchings in dense hypergraphs, and similar results in expander graphs.

Contributed Talk Abstracts

Monday, August 24

Proving Guo's Conjecture on Richardson Tableaux

Sen-Peng Eu * Tung-Shan Fu † Yi-Lin Lee ‡ Yi-Hau Kao §

Abstract

Richardson tableaux, introduced by Karp and Precup in 2025, are a subset of standard Young tableaux that index the irreducible components of Springer fibers equal to Richardson varieties. They proved that the number of Richardson tableaux of size n equals the nth Motzkin number by showing that their generating functions are the same. Later, Guo provided a bijective proof of this result by proving that, under the Robinson–Schensted algorithm, the insertion tableaux of noncrossing involutions on $[n]$ coincide with Richardson tableaux of size n, and these involutions can be identified by Motzkin paths of length n. Let $RT(n,k)$ denote the set of Richardson tableaux of size n with k odd columns, where $n \equiv k \pmod 2$. Guo conjectured the following weighted enumeration of $RT(n,k)$ by the comajor statistics.

$$\sum_{T \in RT(n,k)} q^{\mathsf{comaj}(T)} = q^{\binom{k}{2}} \binom{n}{k}_q C_{\frac{n-k}{2}}(q), \text{ where } C_m(q) \text{ denotes the } m\text{th } q\text{-Catalan number.}$$

In this talk, we present two proofs of this conjecture by viewing Richardson tableaux as Motzkin paths. In the first, we develop an algorithm to construct them from Dyck paths and a set of horizontal steps, where we can explicitly track the comajor statistics. In our second proof, we consider generalized Motzkin paths and identify them with specific permutations. This identification transforms the comajor statistics into the standard major statistics on permutations, allowing us to complete the proof by applying Stanley's shuffle theorem.

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Canonical Ramsey Property for All Triangles

Yijia Fang* Gennian Ge† Yang Shu‡ Qian Xu§ Zixiang Xu¶ Dilong Yang ‡

Abstract

Euclidean Ramsey theory asks if we can find specific geometric structures in colored space. A refined version of this, known as the canonical Ramsey property or Gallai-Ramsey theory, asks: Does every finite coloring of Euclidean space contain a congruent copy of a configuration that is either monochromatic or rainbow(all in different colors)? Formally, a configuration X has the canonical Ramsey property if there exists a dimension n such that for every r and every r-coloring of $\mathbb{R}^n$, there exists a congruent copy of X that is either monochromatic or rainbow.

While this was known for specific configurations, such as acute and right triangles, the question of whether all triangles satisfy this property remained open. In this talk, I will present a solution to this problem posed by Gehér, Sagdeev, and Tóth. We prove that 4-dimensional space is sufficient to guarantee the canonical property for any triangle. The proof introduces a construction obtained by gluing correlated copies of the target triangle. By applying the rotation method, we establish the existence of a large monochromatic sphere. Through a boundary point argument, we obtain another large monochromatic sphere and force the appearance of the required congruent copy.

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Structure of k-Matching-Planar Graphs

Kevin Hendrey* Nikolai Karol† David R. Wood‡

Abstract

We define a simple topological graph G (that is, a graph drawn in the plane such that every pair of edges intersect at most once, including endpoints) to be *k-matching-planar* if for every edge $e \in E(G)$, every matching amongst the edges of G that cross e has size at most k. The class of k-matching-planar graphs is a significant generalisation of many other existing beyond planar graph classes, including k-planar graphs. We prove that every simple topological k-matching-planar graph is a subgraph of the strong product of a bounded treewidth graph and a path. This result qualitatively extends the celebrated Planar Graph Product Structure Theorem of Dujmović, Joret, Micek, Morin, Ueckerdt, and Wood [*J. ACM* 2020] and recent product structure theorems for other beyond planar graph classes. Using this result, we deduce that the class of simple topological k-matching-planar graphs is the broadest class of simple beyond planar graphs in the literature that has numerous attractive structural properties.

The proof of our product structure theorem introduces several tools and results of independent interest. For example, we show that every simple topological k-matching-planar graph admits an edge-colouring with $\mathcal{O}(k^3 \log k)$ colours such that monochromatic edges do not cross. We introduce the concept of weak shallow minors, which generalise shallow minors, a key concept in graph sparsity theory. As a byproduct of our proof techniques, we establish upper bounds on the treewidth of graphs with well-behaved circular drawings that qualitatively generalise several existing results. See [arXiv:2507.22395].

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†Monash University, Melbourne, Australia. Supported by Monash Graduate Scholarship.

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2-limited broadcast domination in cubic graphs

Myungho Choi *   Boram Park†

Abstract

For a graph G, a function $f : V(G) \to \{0, 1, 2\}$ is called a 2-limited dominating broadcast on G if for every vertex u, there exists a vertex v such that $f(v) > 0$ and the distance between u and v in G is at most $f(v)$. The *cost* of f means the value $\sum_{v \in V(G)} f(v)$, and the 2-*limited broadcast domination number* of G, denoted by $\gamma_{b,2}(G)$, is the cost of a 2-limited dominating broadcast on G with minimum cost. Henning, MacGillivray, and Yang (2020) conjectured that $\gamma_{b,2}(G) \leq \frac{|V(G)|}{3}$ for every cubic graph G. In this work, we confirm the conjecture.

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More on the Corner-Vector Construction for Spherical Designs

Kenji Tanino* Tomoki Tamaru† Masatake Hirao‡ Masanori Sawa§

Abstract

This talk presents a full generalization of the classical corner-vector method for constructing weighted spherical designs, which we call the *generalized corner-vector method* (K. Tanino et al., *Algebr. Comb.* 8(5):1387–1414, 2025). While the corner-vector method is attractive for its simplicity of construction, it has the limitation that the generated designs have degree of at most 7, as shown by Bajnok (2007). To overcome this limitation and achieve higher degrees, we introduce a generalization of this method.

Our main results are as follows. First, to investigate whether our generalization can achieve high degrees, we establish a uniform upper bound for the degree of designs obtained by our generalization. The proof employs a hybrid argument from analysis and combinatorics. In particular, we use a well-known result by Xu (1998) concerning the interrelation between spherical designs and simplicial designs, together with the cross-ratio comparison method for Hilbert identities introduced by Nozaki and Sawa (2013). Second, we present interesting examples of designs obtained through an extensive analysis of the conditions for their existence. These include higher-degree designs that successfully push up the maximum degree of the classical corner-vector method. Furthermore, we discuss the surprising phenomenon that two sporadic 7-designs generated by our method are characterized in terms of integral lattices; they are shown to be subsets of certain shells of the Barnes-Wall lattice and the shorter Leech lattice.

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Degree-sequence bounds for independent sets via multivariate local occupancy

Ewan Davies* Juspreet Singh Sandhu† Jaehyeon Seo‡ Brian Tan§

Abstract

Occupancy fractions of the hard-core model encode the expected size of a random independent set. Bounds on the occupancy fractions derive the size and number of independent sets, and in sparse settings, relate to Ramsey-theoretic and coloring questions.

We study lower bounds on the occupancy fraction of the hard-core model via the local occupancy method. For general graphs, we prove a tight lower bound in terms of degree sequences, extending work of Davies, Sandhu, and Tan to the multivariate setting and enlarging the fugacity range. Our approach reformulates a local occupancy condition as a linear program and studies explicit dual solutions using spectral estimates and Laplacian positivity.

In the triangle-free setting, the same framework yields a larger admissible fugacity range for occupancy-fraction lower bounds, giving further progress towards a conjecture of Buys, van den Heuvel, and Kang. More broadly, the method provides a novel perspective to treat irregular graphs with the local occupancy methods.

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Webs and Smooth Components of Two Column Springer Fibers

Mike Cummings*

Abstract

Webs and Springer fibers are separately important objects in representation theory: webs give a diagrammatic calculus for tensor invariants of $\mathfrak{sl}_k$, and the cohomology group of Springer fibers can be used to construct the irreducible representations of the symmetric group. Fung's 1997 thesis gave the first evidence of a connection between $\mathfrak{sl}_2$ webs and Springer fibers, showing that webs naturally index and describe the components of certain "two row" Springer fibers. However, this case is known to be far from generic.

This paper deepens this connection with a similar correspondence in the substantially more complicated "two column" case. In particular, and building on works of Fresse, Melnikov, and Sakas-Obeid, we use webs to give a clean characterization of the smooth components of two column rectangle Springer fibers and a simple description of the geometry of these smooth components. We also show that the Poincaré polynomial of the smooth components is invariant under the natural dihedral action on the corresponding webs.

*University of Waterloo, Waterloo, ON, Canada. Supported by NSERC CGS-D 588999-2024, a University of Waterloo President's Graduate Scholarship, and Oliver Pechenik's NSERC Discovery Grant RGPIN-2021-02391.

Canonical Ramsey theorem for graphs with clean intersections

Matías Azócar Carvajal [*] Ayush Basu [†] Christian Reiher [*] Vojtech Rödl [†]
Giovanne Santos [‡] Mathias Schacht [*]

Abstract

The canonical Ramsey theorem, originating in the foundational work of Erdős and Rado, seeks to find, for a given graph F, a graph G such that every colouring of the edges of G contains a canonically coloured copy of F. Extending results of Nešetřil and Rödl and of Prömel and Voigt on canonical Ramsey properties for ordered graphs, we prove that for any sufficiently connected ordered graph F, there exists an ordered graph G such that G has the canonical Ramsey property for F and any two copies of F in G intersect either in a single vertex, a single edge, or not at all. We refer to this condition on the copies of F as the clean intersection property. As a consequence this allows us to construct canonical Ramsey graphs G for a given ordered graph F enjoying additional structural properties. In particular, G can have the same odd girth and clique number as F, and the copies of F in G are not only induced but also preserve the same distances.

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Partitioning triangle-free planar graphs into a forest and a linear forest

Guanwu Liu* Rongxing Xu†

Abstract

Raspaud and Wang [European J. Combin. 29 (2008)] conjectured that every triangle-free planar graph can be vertex-partitioned into an independent set and a forest. Independently, Kawarabayashi and Thomassen [J. Combin. Theory Ser. B 99 (2009)] also remarked that this might be true, after providing another proof of a result of Borodin and Glebov, showing this result for planar graphs of girth 5. Subsequently, Dross, Montassier, and Pinlou [European J. Combin. 66 (2017)] raised the same question and proved that every triangle-free planar graph can be partitioned into a forest and another forest of maximum degree at most 5. More recently, Feghali and Šámal [European J. Combin. 116 (2024)] improved this bound on the maximum degree to 3. In this note, we further improve the result by showing that every triangle-free planar graph can be partitioned into a forest and a linear forest, that is, a forest of maximum degree at most 2.

*Shandong University of Technology, Zibo, China. Supported by NSFC grant (12401459).

†Zhejiang Normal University, Jinhua, China. Supported by NSFC Grant (12401472).

On the conjecture of Beineke and Henning on independent distance dominating sets

Sonwabile Mafunda* Wayne Goddard†

Abstract

A proper coloring of a graph is called distance-k fall if every vertex is within distance k of a vertex of every color. In this talk, we discuss such colorings and their connection to distance domination. We show that every connected 3-colorable graph of order at least three admits a distance-2 fall 3-coloring. Furthermore, we prove that for each integer $k \geq 2$, every tree of order at least k has a k-coloring in which every vertex is within distance $k-1$ of every color. As a consequence, we settle a conjecture of Beineke and Henning on independent distance-d dominating sets in trees.

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Sharp quantitative stability in Hilbert spaces

Yifan Jing*

Abstract

Grothendieck compactness criterion, and the stability theory from model theory, show that the inner product on the unit ball over Hilbert spaces is stable. We study this phenomenon quantitatively. We prove that the inner product is (k, ϵ)-stable for all $k \geq \exp(\pi/\epsilon)$, and it is not (k, ϵ)-stable for $k \leq \exp(\log 2/\epsilon)$, showing that the growth is necessarily exponential in $1/\epsilon$. This answers a question of Conant. We also analyze how stability scales under nonlinear connectives applied to the inner product, and general binary formulas over Hilbert spaces expanded by unitary or normal operators.

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Functional Inequalities and Random Walks on Increasing Subsets of the Hypercube

Fan Chang* Guowei Sun† Lei Yu‡

Abstract

Ding and Mossel initiated the study of the *random walk on* $\{0,1\}^n$ *censored to a subset* $A \subseteq \{0,1\}^n$, motivated by critical percolation on a square in the hexagonal lattice. For increasing subsets of constant density, they proved an $O(n^3)$ upper bound on the mixing time and asked whether the optimal order should be substantially smaller. Very recently, Fei and Ferreira Pinto Jr. established an $O(n^2)$ upper bound for constant-density increasing sets by proving a restricted Poincaré inequality on such sets. Their argument develops an L^2 functional-analytic framework based on a directed gradient, the associated directed Laplacian, and a monotone heat flow, together with an approximate FKG inequality on increasing subsets. In this talk, we present a short, elementary, and purely combinatorial proof of their Poincaré inequality, yielding the same $O(n^2)$ mixing time bound for censored random walks.

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A quite simple formula on Bruhat order and bigrassmannian permutations

Masato Kobayashi*

Abstract

The speaker found a quite simple formula on Bruhat order and bigrassmannian permutations (M. Kobayashi, Order (2011)). Say $i \in \{1, \ldots, n-1\}$ is a descent of $w \in S_n$ if $w(i) > w(i+1)$. Say w is grassmannian if w has a unique descent. It is **bigrassmannian** if both of w and w^{-1} are grassmannian. Let

$$B(w) = \{v \in S_n \mid v \leq w\}$$

where $\leq$ denotes **Bruhat order** as it plays a significant role in the theory of root systems, crystals and even Schubert calculus. The formula asserts that

$$|B(w)| = \sum_{\substack{i<j \\ w(i)>w(j)}} (w(i) - w(j))$$

as **weighted** enumeration of inversions. For example,

$$|B(3412)| = (3-1) + (3-2) + (4-1) + (4-2) = 8$$

counts all bigrassmannian permutations weakly below 3412 such as 2134, 1324, 1243, 2314, 1342, 1423, 3124 and 3412 itself. Further, the speaker extended this formula to the one for **alternating sign matrices** with the natural extension of Bruhat order (M. Kobayashi, Order, 2020). This surprisingly exhibits harmony with arithmetic of permutations, Coxeter groups and the theory of distributive lattices. In this talk, we explain details of these formulas.

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Perfect tilings with the generalised triangle in k-graphs

Weichan Liu* Xiangxiang Nie† Donglei Yang‡ Lin-Peng Zhang§

Abstract

Denote by T_k the generalised triangle, a k-uniform hypergraph on vertex set $\{1, 2, \ldots, 2k-1\}$ with three edges $\{1, \ldots, k-1, k\}$,$\{1, \ldots, k-1, k+1\}$ and $\{k, k+1, \ldots, 2k-1\}$. Recently, Bowtell, Kathapurkar, Morrison and Mycroft [arXiv: 2505.05606] established the exact minimum codegree threshold for perfect T_3-tilings in 3-graphs. In this paper, we extend their result to all $k \geq 3$, determining the optimal minimum codegree threshold for perfect T_k-tilings in k-graphs. Our proof uses the lattice-based absorption method, as is usual, but develops a unified and effective approach to build transferrals for all uniformities, which is of independent interest. Additionally, we establish an asymptotically tight minimum codegree threshold for a rainbow variant of the problem.

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Generation theorem for 5-connected graphs

Yoshimi Egawa*

Abstract

We consider only finite, undirected, simple graphs with no loops and no multiple edges. For an edge e of a graph G, we let G/e denote the graph obtained from G by contractiong e into a single vertex (and replacing each resulting pair of multiple edges by a simple edge). For an integer $k \geq 2$ and a k-connected graph G, an edge e of G is said to be k-contractible if G/e is k-connected.

It is proved by Tutte that every minimally 3-connected graph other than K_4 has a 3-contractible edge; that is to say, startingwith K_4, one can generate all 3-connected graphs by repeated applications of edge additions and inverse operations of contraction. Similar generation theorems for 4-connected graphs are also known. In this talk, we introduce a generation theorem for 5-connected graphs.

We let $\mathcal{H}_5$ denote the family of 5-connected graphs G which have a vertex v such that $G - v$ is a 4-regular 4-connected graph. Also we consider the following operation: delete a vertex x of a graph G having degree 5, and add three edges (not in $E(G)$) on $N_G(x)$ so that the graph M on $N_G(x)$ formed by the new edges is isomorphic to $P_2 \cup P_3$. We let $G^{(x;M)}$ denote the resulting graph. A vertex x of a 5-connected graph G having degree 5 is said to be 5-liftable if there exists a graph M on $N_G(x)$ with $M \cong P_2 \cup P_3$ and $E(M) \cap E(G) = \emptyset$ such that $G^{(x,M)}$ is 5-connected. The following theorem is the main result in this talk.

Theorem 1. Every minimally 5-connected graph not belonging to $\mathcal{H}_5$ has a 5-contractible edge or a 5-liftable vertex.

If we regard Theorem 1 as a generation theorem for 5-connected graphs, the family $\mathcal{H}_5$, which one starts with, is rather large. However, we also have a generation theorem for 4-regular 4-connected graphs which shows that starting with K_5 and $K_{4,4}$, one can generate all 4-regular 4-connected graphs. Thus one can generate all 5-connected graphs starting with K_6 and $K_1 + K_{4,4}$.

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On supersaturation in the Erdős–Sós problem

Andrey Kupavskii* Yakov Shubin†

Abstract

The following classical question in extremal set theory is due to Erdős and Sós: what is the maximum size of a family $\mathcal{F} \subset \binom{[n]}{k}$ containing no two sets $F_1, F_2 \in \mathcal{F}$ such that $|F_1 \cap F_2| = t$? Frankl and Füredi determined the exact answer, namely $\binom{n-t-1}{k-t-1}$, for $n > n_0(k)$ and $k \geqslant 2t+2$.

In this work, we study a supersaturation version of the Erdős–Sós problem. Given a family $\mathcal{F} \subset \binom{[n]}{k}$ of fixed size ℓ, what is the minimum possible number $\rho(\ell)$ of pairs $F_1, F_2 \in \mathcal{F}$ such that $|F_1 \cap F_2| = t$?

For fixed k, with $k \geqslant 2t+3$, and as $n \to \infty$, we determine the exact threshold at which the minimum number of such pairs matches, up to a constant factor, the expected number in a random ℓ-element family. In particular, we prove that $\rho(\ell) = \Theta\left(\frac{\ell^2}{n^t}\right)$ holds beyond a certain threshold. We also determine the exact value of $\rho(\ell)$ in the case where ℓ does not significantly exceed $\binom{n-t-1}{k-t-1}$.

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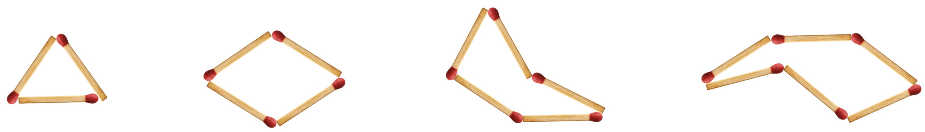
Graphs in Euclidean Ramsey Theory

Maria Axenovich* Dingyuan Liu* Arsenii Sagdeev*

Abstract

Given a graph H, let $\chi_H(\mathbb{R}^n)$ be the smallest positive integer r such that there exists an r-coloring of $\mathbb{R}^n$ with no monochromatic unit-copy of H, that is a set of $|V(H)|$ vertices of the same color such that any two vertices corresponding to an edge of H are at distance one. This Ramsey-type function extends the famous Hadwiger–Nelson problem on the chromatic number $\chi(\mathbb{R}^n) = \chi_{K_2}(\mathbb{R}^n)$ of the space from a complete graph K_2 on 2 vertices to an arbitrary graph H.

Among other results, we show that $\chi_H(\mathbb{R}^n) = \chi(\mathbb{R}^n)$ for any even cycle H of length 8 or at least 12 as well as for any forest and that $\chi_H(\mathbb{R}^n) = \lceil \chi(\mathbb{R}^n)/2 \rceil$ for any sufficiently long odd cycle.



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Chromatic thresholds for linear equations and recurrence

Hong Liu* Zhuo Wu† Ningyuan Yang‡ Shengtong Zhang§

Abstract

Motivated by classical problems in extremal graph theory, we study a chromatic analogue of Roth-type questions for linear equations over $\mathbb{F}_p$. Given a homogeneous equation $\mathcal{L} : \sum_{i=1}^{k} c_i x_i = 0$ with $k \geq 3$, we study $\mathcal{L}$-solution-free sets $A \subseteq \mathbb{F}_p$ through the chromatic number of the Cayley graph $\mathrm{Cay}(\mathbb{F}_p, A)$. We introduce the *chromatic threshold* $\delta_\chi(\mathcal{L})$, the minimum density that guarantees bounded chromatic number of $\mathrm{Cay}(\mathbb{F}_p, A)$ among all $\mathcal{L}$-solution-free sets A, and determine exactly when $\delta_\chi(\mathcal{L}) = 0$. We prove that $\delta_\chi(\mathcal{L}) = 0$ if and only if $\mathcal{L}$ contains a zero-sum subcollection of at least three coefficients.

A key ingredient is a quantitative chromatic lower bound for Cayley graphs on $\mathbb{Z}_p^n$ generated by Hamming balls around the all-ones vector. This is obtained by introducing a new Kneser-type graph that admits a natural embedding into $\mathbb{Z}_p^n$, together with an equivariant Borsuk–Ulam type argument. As a consequence, we resolve a question of Griesmer. We further relate our classification to the hierarchy of measurable, topological, and Bohr recurrence. In particular, we show that every infinite discrete abelian group admits a set that is topological recurrent but not measurable recurrent, extending the seminal examples of Kříž and Ruzsa.

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Intersections of Schubert varieties and smooth T-stable subvarieties of flag varieties

Jaehyun Hong* Eunjeong Lee† Seonjeong Park‡

March 12, 2026

Abstract

Let G be a reductive algebraic group over $\mathbb{C}$ and B a Borel subgroup. Hessenberg varieties are subvarieties of the flag variety G/B that provide a fruitful connection between geometry, representation theory, and combinatorics. More specifically, the Weyl group W of G acts on the cohomology of a regular semisimple Hessenberg variety. In type A, the study of this representation is related to the Stanley–Stembridge and Shareshian–Wachs conjectures on chromatic symmetric functions.

In this talk, we study a basis for the cohomology of a regular semisimple Hessenberg variety arising from the Białynicki-Birula decomposition. We first investigate the irreducibility and smoothness of the closures of Białynicki-Birula cells in regular semisimple Hessenberg varieties, called Hessenberg Schubert varieties. We describe these properties in terms of the regularity of a certain graph, which yields a sufficient condition for a Hessenberg Schubert variety to be smooth. We then study the admissible basis elements that generate the cohomology as a W-module. This talk is based on joint work with Jaehyun Hong and Seonjeong Park.

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Optimal and efficient partite decompositions of hypergraphs

Andrew Krapivin[*] Benjamin Przybocki[†] Nicolás Sanhueza-Matamala[‡]
Bernardo Subercaseaux[§]

Abstract

We study the problem of decomposing the edges of an n-vertex d-uniform hypergraph H into a family of edge-disjoint complete d-partite hypergraphs (d-cliques). We show that there is a family F in which every vertex of H belongs to at most $(\frac{1}{d!} + o_d(1))n^{d-1}/\log_2 n$ members of F. This settles the central question of a line of research initiated by Erdős and Pyber (1997) for graphs, and more recently by Csirmaz, Ligeti, and Tardos (2014) for hypergraphs. The $d = 2$ case of this theorem answers a 40-year-old question of Chung, Erdős, and Spencer (1983).

The search for d-clique decompositions is motivated by applications to theoretical computer science, such as data compression and speed-up of algorithms. Our proof is constructive, and the d-clique decompositions can be found by an efficient and simple algorithm, improving upon results by Feder and Motwani (1995) and Mubayi and Turán (2010).

Our methods allow us to obtain optimal biclique decompositions in dense random graphs, addressing other question of Chung, Graham and Erdős. We also obtain improved bounds in graph families of polynomially bounded shattering, or bounded VC-dimension.

[*]Carnegie Mellon University, Pittsburgh, Pennsylvania, USA. Supported by Jeanne B. and Richard F. Berdik ARCS Pittsburgh Endowed Scholar Award.

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The existence of 2-factors in $\frac{3}{2}$-tough maximal planar graphs

Lili Hao*

Abstract

The toughness of a graph G is defined as the minimum value of $|S|/c(G-S)$ over all cutsets S of G if G is noncomplete, and is defined to be ∞ if G is complete. For a real number t, we say that G is t-tough if its toughness is at least t. Followed from the classic 1956 result of Tutte, every more than $\frac{3}{2}$-tough planar graph on at least three vertices has a 2-factor. In 1999, Owens constructed a sequence of maximal planar graphs with toughness $\frac{3}{2}-\varepsilon$ for any $\varepsilon > 0$, but the graphs do not contain any 2-factor. He then posed the question of whether there exists a maximal planar graph with toughness exactly $\frac{3}{2}$ and with no 2-factor. This question was recently answered affirmatively by Shan. This naturally leads to the question: under what conditions does a $\frac{3}{2}$-tough maximal planar graph contain a 2-factor? In this talk, we provide a sufficient condition for the existence of 2-factors in $\frac{3}{2}$-tough maximal planar graphs, stated as a bound on the distance between vertices of degree 3.

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Coloring of generalized signed graphs and its application to DP coloring

Ligang Jin* Yingli Kang† Xuding Zhu‡

Abstract

The concept of DP-coloring of graphs was introduced by Dvořák and Postle in 2018, and was used to prove that planar graphs without cycles of length from 4 to 8 are 3-choosable, which confirms a conjecture of Borodin. In the same paper, they proposed a more natural and stronger claim that such graphs are DP-3-colorable. Recently, we confirms that claim by proving a stronger result that planar graphs having no cycle of length 4, 6 or 8 are DP-3-colorable. In this talk, we will present the proof of this result. The proof uses the concept of coloring of generalized signed graphs, which was introduced by Tsai-Lien Wong and two of us in 2021. We show that the statement of DP-coloring can be reformulated in the language of coloring of generalized signed graph. Then in the setting of coloring of generalized signed graph, we can successfully apply many graph operations for the proof of reducible configurations, not only deleting vertices or identifying two vertices, but also adding signed edges, identifying more pairs of vertices and their combination. The reducible configurations are constructed and confirmed in a systematical way. We show that for many reducible configurations, both their I_k-extension and J_k-extension are reducible as well. The proof is completed by using standard discharging method. For more details on the result, please see the paper (arXiv:2412.19059v1).

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Many Antipodal Pairs Force many Neighboring Pairs

Gábor Damási* Laurenţiu Ploscaru †

Consider a finite set $X = \{x_1, \ldots, x_n\} \subset \mathbb{R}^2$ of diameter 1. In this talk we investigate the relation between the number of pairs of points in X that are close to each other and the number of pairs of points in X whose distance is close to 1.

More precisely, for $0 < \varepsilon < 1$, we call a pair (x_i, x_j) ε-*antipodal* if $\|x_i - x_j\| \geq 1 - \varepsilon$ and let $\mathcal{A}_\varepsilon(X)$ denote the set of antipodal pairs in X, whereas a pair (x_i, x_j) is ε-*neighboring* if $\|x_i - x_j\| \leq \varepsilon$ and let $\mathcal{N}_\varepsilon(X)$ denote the set of neighboring pairs. We are interested in the ratio $R_\varepsilon(X) := |\mathcal{N}_\varepsilon(X)|/|\mathcal{A}_\varepsilon(X)|$. The study of this quantity was recently initiated by Steinerberger, who showed that $R_\varepsilon(X) = \Omega\left(\sqrt[4]{\varepsilon^3/\log(\varepsilon^{-1})}\right)$. He also provided examples where the ratio is of order $\varepsilon^{1/2}$ and asked whether this is optimal. Recently Korsky showed that $R_\varepsilon(X) = \Omega\left(c_2\sqrt{\varepsilon/\log(\varepsilon^{-1})}\right)$.

We show that the $\varepsilon^{1/2}$ bound is indeed optimal, i.e., there are universal constants $c_1, c_2 > 0$ for which $R_\varepsilon(X) \geq c_2\sqrt{\varepsilon}$ whenever $|X| \geq c_1 \cdot \varepsilon^{-2}$. Following previous ideas, we proceed by covering X with boxes of small diameter and define an auxiliary *box graph* on these boxes by connecting pairs that are at least $1 - \varepsilon$ far from each other. The problem then reduces to bounding the largest eigenvalue of this graph. We do so via a key geometric lemma which shows that the box graph satisfies a natural sparsity condition. Our result was obtained through a close collaboration with the AI software ChatGPT 5.4.

*HUN-REN Alfréd Rényi Institute of Mathematics, ELTE Eötvös Loránd University, Budapest, Hungary. Supported by ERC no. 882971 GeoScape, by the János Bolyai Research Scholarship of the Hungarian Academy of Sciences, the STARTING-25 project no. 152199 and the EXCELLENCE-24 project no. 151504.

†HUN-REN Alfréd Rényi Institute of Mathematics, Budapest. Supported by ERC no. 882971 GeoScape and ERMiD no. 101054936.

Multiplicative irreducibility of shifted multiplicative subgroups

Seoyoung Kim * Chi Hoi Yip† Semin Yoo‡

Abstract

A central theme in additive combinatorics is the interplay between addition and multiplication. Roughly speaking, sets with strong multiplicative structure are not expected to exhibit rich additive structure, and vice versa. In a recent breakthrough, Kalmynin resolved conjectures of Lev–Sonn and Sárközy on additive decompositions of multiplicative subgroups of prime fields. Motivated by this work, we investigate multiplicative analogues of these questions, including questions raised by Sárközy. More precisely, let p be an odd prime, let $\mathbb{F}_p$ be the finite field with p elements, let G be a proper subgroup of $\mathbb{F}_p^*$, and let $\lambda \in G$. We show that $(G-\lambda)\setminus\{0\}$ cannot be written as a product set AB with $A, B \subseteq \mathbb{F}_p^*$ and $|A|, |B| \geq 2$. We also show that a shifted multiplicative subgroup cannot be written as a ratio set A/A. Our proofs combine Stepanov's method with techniques involving symmetric polynomials.

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Anticoncentration of random spanning trees in almost regular graphs

Hyunwoo Lee*

Abstract

The celebrated formula of Otter *[Ann. of Math. (2) 49 (1948), 583–599]* asserts that the complete graph contains exponentially many non-isomorphic spanning trees. In this paper, we show that every connected almost regular graph with sufficiently large degree already contains exponentially many non-isomorphic spanning trees. Indeed, we prove a stronger statement: for every fixed n-vertex tree T,

$$\Pr\big[\mathcal{T} \simeq_{\text{iso}} T\big] = e^{-\Omega(n)},$$

where $\mathcal{T}$ is a uniformly random spanning tree of a connected n-vertex almost regular graph with sufficiently large degree. To prove this, we introduce a graph-theoretic variant of the classical balls–into–bins model, which may be of independent interest.

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Exact minimum co-degree bounds for tight Hamilton cycles

Richard Lang* Shoham Letzter† Arjun Ranganathan†
Nicolás Sanhueza-Matamala‡

Abstract

One of the earliest hypergraph extensions of Dirac's theorem is a conjecture of Katona and Kierstead from 1999, which asserts that for all uniformities $k \geq 3$, any n-vertex k-graph in which every $(k-1)$-set extends to at least $\frac{n-k+2}{2}$ edges contains a tight Hamilton cycle. Rödl, Ruciński and Szemerédi first proved this conjecture asymptotically in 2008, showing that $n/2 + o(n)$ suffices for any uniformity, and obtained an exact bound for $k = 3$ in 2011.

We establish a complete resolution of this conjecture for all $k \geq 3$, improving the above asymptotic bounds to exact ones. Our proof involves a range of structural arguments for near-extremal hypergraphs and utilises a recent Hamiltonicity framework of Lang and Sanhueza-Matamala in the non-extremal regime that avoids traditional approaches using regularity.

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A constructive characterization of uniformly 4-connected graphs

Xiang Chen*

August 2026

Abstract

A graph is said to be *uniformly k-connected* if between any pair of vertices the maximum number of internally-disjoint paths is exactly k. In this talk, we will give a constructive characterization of the class of uniformly 4-connected graphs. The characterization is based on the application of graph operations to appropriate vertex and edge sets in uniformly 4-connected graphs, that is, any uniformly 4-connected graph can be obtained from C_5^2 or C_6^2 by a number of Δ_1^+ or Δ_2^+-operations to quasi 4-compatible sets.

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(Fractional) balanced coloring of signed graphs

Xiaowei Yu*

Abstract

Signed graphs are a more comprehensive model for networks where there are two types of competing relations such as animosity and friendship. A signed graph (G, σ) is a graph $G = (V, E)$ together with a signature function $\sigma : E(G) \to \{-, +\}$ which assigns to each edge e a sign $\sigma(e)$. Given a signed graph (G, σ), a balanced (p, q)-coloring of (G, σ) is an assignment f of q colors to each vertex from a pool of p colors such that each color class induces a balanced set. The balanced chromatic number is the minimum number of balanced subsets of vertices which cover $V(G)$, which is a refinement of the classic chromatic number of graphs, providing a better platform to study the correlation of minor theory and coloring of graphs.

In this talk, I shall give an overview of (fractional) balanced coloring of signed graphs and present our results.

*IBS ECOPRO, Daejeon, Korea. Supported by NSFC (No. 12571367).

Covering integer points of a cross-polytope by subspaces

Nóra Frankl * Dmitry Gribanov † Maxim Klimenko‡ Andrey Kupavskii§

Abstract

Let $C_r = \{||x||_1 \leq r\}$ be an ℓ_1-ball of radius $r > 1$ around the origin, and let $g(C_r)$ denote the minimum number of proper hyperplanes required to cover the set $\mathbb{Z}^d \cap C_r$. In 2002, Bárány, Harcos, Pach, and Tardos proved that

$$g(C_r) \leq 2^{(1+o(1))d} \cdot r^{\frac{d}{d-1}},$$

which has the correct order of magnitude as a function of r. We improve the dependence on the dimension d in this bound by relating the covering problem to sphere packing densities in $\mathbb{R}^d$. In particular, we show that

$$g(C_r) \leq (2^{(1+o(1))d} \cdot \delta) \cdot r^{\frac{d}{d-1}},$$

where δ denotes the optimal sphere packing density in dimension d. It is known that

$$2^{(-1+o(1))d} \leq \delta \leq 2^{(-0.599\ldots+o(1))d}.$$

In particular, if the packing density is close to lower bound, our estimate yields a substantially improved dependence on the dimension, reducing the leading term to $2^{o(d)}$. Even with the best currently known upper bounds on δ, our result already improves the exponential factor to approximately $2^{0.401d}$.

Additionally, we provide an algorithm for constructing an integer point $x \in \mathbb{Z}^d$ that avoids a given family of subspaces $\mathcal{S}$, i.e., $x \notin S$ for all $S \in \mathcal{S}$, with

$$\|x\|_1 = O\Big(|\mathcal{S}|^{\frac{d-1}{d}}\Big).$$

The algorithm runs in time $O\big(|\mathcal{S}|(d + |\mathcal{S}|)\big)$.

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Room F

Monday, August 24 | 17:00–17:30

Simultaneous popular polynomial Szemerédi theorem over finite fields

David Conlon* Dingding Dong† Guo-Dong Hong‡

Abstract

A celebrated result in ergodic Ramsey theory is Bergelson and Leibman's polynomial Szemerédi theorem, which asserts that any subset of the integers with positive upper density must contain arbitrarily long polynomial progressions. The proof relies on the study of recurrence properties for the multiple polynomial ergodic averages of dynamical systems via the Furstenberg correspondence principle.

Bergelson and Leibman's result was subsequently strengthened in two directions. In one direction, Frantzikinakis and Kuca proved a multiple polynomial recurrence theorem with Khintchine-type lower bounds. In the other direction, Lyall and Magyar initiated the study of simultaneous polynomial recurrence for a single transformation.

In this talk, we further investigate the phenomenon of simultaneous polynomial recurrence over finite fields. To be more precise, let $k \geq 2$ and let $P_1, \ldots, P_k \in \mathbb{Z}[t]$ be linearly independent polynomials over $\mathbb{Q}$ with $P_i(0) = 0$. We prove that for every $\epsilon > 0$, there exists $p_0 = p_0(\epsilon, P_1, \ldots, P_k)$ such that for every prime $p \geq p_0$ and every $A \subset \mathbb{F}_p$, there exists $d \in \mathbb{F}_p^\times$ for which

$$\mathbb{E}_{x \in \mathbb{F}_p} \mathbf{1}_A(x) \prod_{i=1}^{k} \mathbf{1}_A\big(x + P_i(d)\big)^{\omega_i} \geq \alpha^{\,1+\sum_i \omega_i} - \epsilon$$

holds simultaneously for all $\omega = (\omega_1, \cdots, \omega_k) \in \{0,1\}^k$, where $\alpha = \mathbb{E}_{x \in \mathbb{F}_p} \mathbf{1}_A(x)$.

The proof combines a quantitative equidistribution result for polynomial sequences on low-rank Bohr sets with an application of Peluse's polynomial Szemerédi theorem over finite fields.

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Contributed Talk Abstracts

Tuesday, August 25

Coxeter Condorcet domains and Condorcet root posets

Yibo Gao* Yulin Peng†

Abstract

Condorcet domains are subsets of permutations that ensure pairwise majority voting yields acyclic outcomes, and they are an active area of research in social choice theory. In this paper, we extend the theory of Condorcet domains (as recently formalized by Puppe et al., 2024) to the broader setting of arbitrary finite Coxeter groups. The core contribution of our approach is the introduction of Condorcet root posets, defined on root systems of Coxeter groups. By establishing natural morphisms between Condorcet domains and these posets, we develop a unified language that efficiently characterizes a wide range of desirable properties of Condorcet domains, such as being closed, maximal, connected, peak-pit, of maximal-width, and of tiling-type. Notably, we establish an exact one-to-one correspondence between closed Condorcet domains and Condorcet root posets, a result that is novel even in the classical type A setting (in the symmetric group). Through this structural lens, we extend the median graph representation of closed Condorcet domains (Puppe & Slinko, 2019) to arbitrary finite Coxeter groups, demonstrating that it can be characterized by the skeleton of the associated Condorcet root posets. Moreover, we strengthen and generalize several classical results (can be found in Puppe et al., 2024 and Reiner et al., 2025) as follows. We establish that closed and connected domains are necessarily peak-pit; we prove that the tiling-type property is equivalent to the conjunction of being maximal, of maximal-width, and peak-pit; and we conclude that strictly positive voting profiles on connected Condorcet domains yield outcomes with only simple ties.

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Spectral Turán Problems for Expansion Hypergraphs

Zhenyu Ni* Dongquan Cheng† Jing Wang‡ Liying Kang§

Abstract

Given a graph F, the expansion $F^{(r)}$ of F is defined as the r-uniform hypergraph obtained from F by adding a set of $(r-2)$ distinct new vertices to each edge of F. We investigate spectral stability results for hypergraphs and their applications.

Firstly, we establish a spectral stability property: for any r-uniform hypergraph containing no copy of the expansion $F^{(r)}$ of a $(k+1)$-chromatic graph F, if its p-spectral radius is close to the extremal value, then the hypergraph is structurally close to $T_r(n,k)$, the complete k-partite r-uniform hypergraph on n vertices whose partition sizes differ by at most one.

Based on the above spectral stability result, we characterize the unique extremal hypergraph that maximizes the p-spectral radius among all n-vertex r-uniform hypergraphs containing no $tK_{k+1}^{(r)}$, that is, no t vertex-disjoint copies of the expansion $K_{k+1}^{(r)}$ of K_{k+1}. We prove that this extremal hypergraph is isomorphic to $K_{t-1}^r \vee T_r(n-t+1,k)$, namely, the join of the complete r-uniform hypergraph K_{t-1}^r and $T_r(n-t+1,k)$.

As a corollary, we conclude that $K_{t-1}^r \vee T_r(n-t+1,k)$ is the unique extremal hypergraph for $tK_{k+1}^{(r)}$, which extends a result of Pikhurko [J. Combin. Theory Ser. B, 103 (2013) 220–225] for expanded complete graphs.

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Small Quasi-Kernel in Digraphs with Bounded Out-Degree

Alexander Clow *

Abstract

In a digraph D, a quasi-kernel is an independent set Q such that for every vertex u, there is a vertex $v \in Q$ satisfying $\text{dist}(v, u) \leq 2$. In 1974 Chvátal and Lovász showed every digraph contains a quasi-kernel. In 1976, P. L. Erdős and Székely conjectured that every sourceless digraph has a quasi-kernel of order at most $\frac{n}{2}$. If correct, this bound is optimal even for 1-out-regular digraphs as evidenced by the directed 4-cycle. Despite significant recent attention by the community the problem remains far from solved, with no bound of the form $(1-\epsilon)n$ known. By defining a new greedy algorithm for constructing quasi-kernels we prove that if D is sourceless with maximum out-degree 3, then D has quasi-kernel of order at most $4n/7$. Additionally, by using an averaging argument together with this algorithm, we show that all sourceless orientations D of a graph G with girth at least 7 have a quasi-kernel of order at most $\frac{(d^2+4)n}{(d+2)^2}$, where d is the maximum out-degree of D.

*Simon Fraser University, Burnaby, Canada. Supported by the Natural Sciences and Engineering Research Council of Canada (NSERC) through PGS D-601066-2025.

List colouring of toroidal grids

Chenglong Deng* Yangyan Gu† Xuding Zhu‡

Abstract

Combinatorial Nullstellensatz is a powerful tool in the study of list colouring of graphs. To prove that a graph G has choice number $\mathrm{ch}(G)$ at most k, it suffices to prove that G has Alon-Tarsi number $AT(G)$ at most k. Dvořák proved a strengthening of Combinatorial Nullstellensatz, which shows that the coefficients of monomials in the expansion of the graph polynomial P_G contain more information about list colouring of G than its Alon-Tarsi number. It says that if L is a list assignment of G, and $\rho : V(G) \to \mathbb{Z}$ satisfies $\rho(v) < |L_v|$ for all v and $\sum_{v \in V(G)} \rho(v) = |E(G)| - 1$ and G is not L-colorable, then for every colour c, $\sum_{v \in V(G)} [x^{\rho + 1_v}]\, P_G \;\cdot\; \chi_{L,c}(v) = 0$. This result has the potential to prove $ch(G) \leq k$ even if $AT(G) > k$. Dvořák gave a few small example graphs for this to be the case. However, applying Dvořák's result involves complicated calculations, and it is a challenging task to apply this result to some interesting infinite families of graphs. In this paper, we apply Dvořák's result to the study of list colouring of toroidal grids. It is known that for any odd positive integers ℓ, m, $T_{\ell,m} = C_\ell \square C_m$ has Alon-Tarsi number 4. On the other hand, it is conjectured that these graphs have choice number 3. This family of graphs seems to be a perfect candidate for the application of Dvořák's result. We formulate a conjecture on the rank of a matrix whose entries are $[x^\rho]P_G$ for various monomials x^ρ of P_G, where $G = T_{\ell,m}$. Using Dvořák's result, we prove that this conjecture implies that $T_{\ell,m}$ is 3-choosable. We verify this conjecture for the case $\ell = 3$. As a consequence, the toroidal grids $T_{3,m}$ are 3-choosable for all odd integers m.

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Lower bound theorems on the numbers of faces of polytopes with at most $3d-1$ vertices

Guillermo Pineda Villavicencio * Jie Wang † David Yost ‡

Abstract

Let P be a d-dimensional polytope (in short, d-polytopes) with $d+s$ vertices. For $1 \leq s \leq d$, Grünbaum conjectured that the function

$$\theta_k(d+s,d) := \binom{d+1}{k+1} + \binom{d}{k+1} - \binom{d+1-s}{k+1}, \tag{1}$$

gives the minimum number of k-faces of a d-polytope with $d+s$ vertices, for each $k \in [1,\ldots,d-1]$. This was confirmed and completely proved by Xue in 2021; the minimising polytopes all have $d+2$ facets. For $s = d+1, s = d+2$, the corresponding minimum has been found in several papers. Since d-polytopes with exactly $d+2$ facets are well understood, we only need to look at d-polytopes with at least $d+3$ facets. We give a refined lower bound result of d-polytopes with at most $2d$ vertices and at least $d+3$ facets, and then present our main result:

Let P be a d-polytope with $2d+s$ vertices (the dimension and the number of vertices are fixed) and at least $d+3$ facets, $1 \leq s \leq d-1$. Then

$$\eta_k(2d+s,d) := \binom{d+1}{k+1} + 2\binom{d}{k+1} - \binom{d-s}{k+1}$$

gives the minimum number of k-faces of P, for each $k \in [1,\ldots,d-2]$.

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Intersecting families of spanning trees in complete bipartite graphs

Gordian Bruns* Dheer Desai† Alexander Gavrilyuk‡ Josias Gomez§
Nathan Lindzey¶

Abstract

Two spanning trees of a graph are t-intersecting if they share a forest of t edges. A family of spanning trees is t-intersecting if any two members are t-intersecting.

We prove an Erdős-Ko-Rado-type result for t-intersecting families of spanning trees of a complete bipartite graph $K_{n,n}$. This is a counterpart to recent work by Frankl et al. [arXiv:2502.08128 (2025)], where they considered intersecting families of spanning trees in complete graphs. Their proof uses the spread approximation technique [arXiv:2309.00097 (2023)] in conjunction with the Lopsided Lovász Local Lemma. While our proof is also based on the spread approximation, it required a different approach, which may also provide an alternative argument for [arXiv:2502.08128 (2025)].

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MacNeille completions of parabolic quotients in the symmetric group

Yibo Gao* Hanlin Xu†

Abstract

Alternating sign matrices (ASMs) arise as the Dedekind–MacNeille completion of the Bruhat order on the symmetric group. They enjoy fruitful combinatorial and geometric properties, with a particularly rich history on enumerations and bijections. In this paper, we explicitly describe the Dedekind-MacNeille completion of the Bruhat order on any parabolic quotients of the symmetric group. It is naturally a subposet of the alternating sign matrices, with different lattice operations. Moreover, we demonstrate the relations between the meet and join operations in this lattice with taking unions and intersection of the corresponding ASM varieties. Finally, we conclude with a more detailed discussion of special cases.

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Improving $R(3,k)$ in just two bites

Zion Hefty* Paul Horn† Dylan King‡ Florian Pfender§

Abstract

The Ramsey number $R(t,k)$ is the smallest n such that any red-blue edge coloring of the n-vertex complete graph has either a t-vertex red complete subgraph or a k-vertex blue complete subgraph. We will investigate the history of asymptotic bounds on the extreme off-diagonal Ramsey numbers $R(3,k)$ and present a new lower bound that has been conjectured to be asymptotically tight.

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The structure of $\Delta(1,2,2)$-free tournaments

Seokbeom Kim* Taite LaGrange† Mathieu Rundström‡ Arpan Sadhukhan§ Sophie Spirkl¶

Abstract

Let $\Delta(1,2,2)$ be the tournament obtained from the cyclic triangle by substituting two-vertex tournaments for two of its vertices. In this talk, we present a complete structural description of tournaments excluding $\Delta(1,2,2)$ as a subtournament. Specifically, we prove that such tournaments either belong to one of the basic classes for which we have explicit descriptions, or admit one of two decompositions. This decomposition scheme can be further translated into a construction for such tournaments and provides a theorem describing their global structure. As a corollary, we obtain tight bounds on several tournament parameters for this class of tournaments, such as the dichromatic number and the maximum size of a transitive subtournament.

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¶University of Waterloo, Waterloo, Ontario, Canada. We acknowledge the support of the Natural Sciences and Engineering Research Council of Canada (NSERC), [funding reference number RGPIN-2020-03912]. Cette recherche a été financée par le Conseil de recherches en sciences naturelles et en génie du Canada (CRSNG), [numéro de référence RGPIN-2020-03912]. This project was funded in part by the Government of Ontario. This research was conducted while Spirkl was an Alfred P. Sloan Fellow.

Between proper and square colorings of planar graphs with maximum degree at most four

Xujun Liu[*] Zihui Xu[†] Xin Zhang[‡]

Abstract

An i-independent set is a vertex set whose pairwise distance is at least $i+1$. A proper (square) k-coloring of a graph G is a partition of its vertex set into k independent (2-independent) sets. A packing $(1^j, 2^k)$-coloring of a graph G is a partition of $V(G)$ into j independent sets and k 2-independent sets. It can be viewed as intermediate colorings between proper and square coloring. Wegner conjectured in 1977 that every planar graph with maximum degree at most four is square 9-colorable. Bousquet, Deschamps, de Meyer, and Pierron proved an upper bound of 12, which is the current best result toward the conjecture of Wegner. In this paper, we prove two analogue results that every planar graph with maximum degree at most four is packing $(1, 2^{10})$-colorable and packing $(1^2, 2^7)$-colorable. We also provide a construction for the sharpness of our results.

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Decomposability of polyhedra and their graphs

Debra Briggs* Krzysztof Przesławski† David Yost‡

Abstract

We use graph theoretic methods to study the Minkowski decomposability of polytopes.

A compact convex set is called decomposable if it can be expressed as the sum of two other sets which are not homothetic to it. For example, the cube can be decomposed as the sum of three line segments. The closed unit disc in the plane is decomposable as the sum of a Reuleaux triangle and its reflection about its centroid. On the other hand, any triangle or simplex is indecomposable.

We restrict our attention in this talk to 3-dimensional polyhedra. Our aim is to make explicit the classification (up to combinatorial equivalence) of polyhedra with 16 edges, and then to determine their decomposability status. The corresponding problem for polyhedra with up to 15 edges was solved some years ago by various authors. We build on the techniques of M. Kallay, who showed that the question of decomposability can be reduced to properties of the skeleton of the polyhedron (i.e. the graph of its vertices and edges). We survey a number of conditions (some of them new) on the skeleton of a polyhedron which are sufficient for its decomposability or indecomposability. We highlight several graphs, in particular K_4 and $K_{2,3}$, whose presence in the skeleton helps to decide decomposability. In many cases, but not all, the combinatorial type of a polyhedron determines whether or not it is decomposable. For the 296 types of polyhedra with 9 faces and 9 vertices, we show that 234 of them are combinatorially indecomposable, and 49 are combinatorially decomposable. We pay particular attention to the remaining 13 examples, which are ambiguous in the sense that they admit both indecomposable and decomposable geometric realizations.

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Ellipsoidal designs and the Prouhet–Tarry–Escott problem

Hideki Matsumura* Masanori Sawa†

Abstract

The notion of ellipsoidal design was first introduced by Pandey (Ramanujan J., 58(4), 2022) as a full generalization of spherical designs on the unit circle S^1. In this talk, we establish connections between ellipsoidal design and the two-dimensional Prouhet–Tarry–Escott problem (PTE_2), originally introduced by Alpers and Tijdeman (J. Number Theory, 123(2), 2007) as a natural generalization of the classical one-dimensional PTE problem (PTE_1). We first provide a combinatorial criterion for constructing solutions of PTE_2 from a pair of ellipsoidal designs. We then give an arithmetic proof of the Stroud-type bound for ellipsoidal designs, and present a classification theorem for designs with equality. Such a classification result is closely related to an open question on the existence of rational spherical 4-designs on S^1, discussed in Cui, Xia and Xiang (Adv. Math., 352, 2019). As far as the speaker know, a solution due to Alpers and Tijdeman is the first and the only known parametric ideal solution of degree 5 for PTE_2. Moreover, as one of our main theorems, we describe the equivalence between the Alpers–Tijdeman solution and a certain two-dimensional extension of the classical Borwein solution for PTE_1. As a consequence, we discover a family of ellipsoidal 5-designs among the Alpers–Tijdeman solution. This talk is based on our paper (Ramanujan J., 68(96), 2022).

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Erdős–Rogers function on hypergraphs

Longma Du* Xinyu Hu† Ruilong Liu‡ Guanghui Wang§

Abstract

Extensive study of off-diagonal Ramsey numbers since 1935 has led to a wealth of classic results and, in recent years, several important breakthroughs. As a natural outgrowth of this line of inquiry, the Erdős–Rogers function, which can be viewed as a dual problem and an extension of the classical Ramsey theory, has likewise received extensive attention. To be precise, for $k \leq t < s < N$, the Erdős–Rogers function $f_{t,s}^{(k)}(N)$ denotes the largest m such that every $K_s^{(k)}$-free k-graph on N vertices contains a $K_t^{(k)}$-free induced subgraph on m vertices. Here, we present an overview of recent developments concerning the Erdős–Rogers function.

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†Shandong University, Jinan, China. Supported by the National Postdoctoral Fellowship Program (C-tier) (GZC20252005).

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Digraphs with density maximized by transitive tournaments

Igor Balla *† Bartłomiej Kielak *† Daniel Král' ‡† Matjaž Krnc §
Filip Kučerák *†

Abstract

A classical theme in extremal graph theory is to determine which host graphs minimize or maximize the homomorphism density of a fixed graph H, possibly given additional constraints. For example, the famous conjecture of Sidorenko and Erdős-Simonovits asserts that the density of every bipartite graph H is asymptotically minimized by quasirandom graphs among all graphs with the same edge density. We are interested in the directed setting with tournaments as host graphs. In this setting, Fox et al. [arXiv:2402.08418] has recently studied the cases where quasirandom tournaments are extremal. We focus on the cases where transitive tournaments are extremal.

Our main result reads as follows: if a directed graph H admits a homomorphism to the directed edge, then among all tournaments on n vertices the number of homomorphisms from H is maximized by the n-vertex transitive tournament. The proof relies on the careful analysis of relationships between homomorphism sets $\mathrm{Hom}(H,T)$ over all possible tournaments T. We also consider a rainbow variant where each edge of H is decorated with a tournament on the same vertex set and we count homomorphisms respecting the decorations. Under the assumptions that H is an orientation of an acyclic graph that admits a homomorphism to the directed edge, we show that the maximum number of such rainbow homomorphisms is attained when all tournaments in the family coincide with the same transitive tournament.

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Planar graphs of odd girth 7 are fractional 14/5-colorable

Xiaolan Hu* Zhouningxin Wang†

Abstract

The fractional chromatic number of a graph G, denoted by $\chi_f(G)$, is defined as the infimum ratio a/b such that G admits a fractional $(a\colon b)$-coloring. We say a graph G is fractionally $(a\colon b)$-colorable if $\chi_f(G) \leq a/b$. In this paper, we study fractional colorings of graphs of odd girth at least 7 and with maximum average degree less than 14/5. We prove that every graph in this family admits a fractional $(14k\colon 5k)$-coloring for some k with an additional constraint on each pair of vertices at distance at least 3. As a consequence, we obtain, for the first time, an upper bound strictly below 3, namely 14/5, for the fractional chromatic numbers of planar graphs of odd girth at least 7.

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Normal 4-pseudomanifolds with a relative 2-skeleton

Biplab Basak[*], Mangaldeep Saha[†], Sourav Sarkar[‡]

Abstract

Face-number invariants are fundamental in combinatorial topology, with the invariant g_2 playing a central role. For a normal d-pseudomanifold K ($d \geq 3$), it is known that $g_2(K) \geq g_2(\mathrm{lk}(v, K))$ for every vertex v. Moreover, if K has at most two singular vertices and satisfies $g_2(K) = g_2(\mathrm{lk}(t, K))$ at a singular vertex t, then $g_3(K) \geq g_3(\mathrm{lk}(t, K))$. We call K *g_2- and g_3-optimal* if $g_2(K) = g_2(\mathrm{lk}(t, K))$ and $g_3(K) = g_3(\mathrm{lk}(t, K))$, for some singular vertex t.

In this talk, we establish structural results for normal 4-pseudomanifolds under g_2- and g_3-optimality conditions. We prove that if K is a normal 4-pseudomanifold with exactly one singular vertex t and is g_2- and g_3-optimal at t, then K can be obtained from the boundary complexes of 5-simplices via a sequence of vertex foldings and connected sum operations.

If K has exactly two singular vertices and is g_2- and g_3-optimal at one of them, then K can be derived from the boundary complexes of 4-simplices through a sequence of one-vertex suspensions, vertex foldings, and connected sums.

Moreover, in this two-singularity case, we provide an alternative characterization. We prove that such a complex K can also be obtained from the boundary complexes of 5-simplices via a sequence of vertex foldings, edge foldings, and connected sums.

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[†]Indian Institute of Technology Delhi, New Delhi, India. Supported by the institute fellowship at IIT Delhi, India.

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Spanning trees of bounded degree in random geometric graphs

Michael Anastos[*] Sahar Diskin[†] Dawid Ignasiak[‡] Lyuben Lichev[§] Yetong Sha[¶]

Abstract

We determine the sharp threshold for the containment of all n-vertex trees of bounded degree in random geometric graphs with n vertices. This provides a geometric counterpart of Montgomery's threshold result for binomial random graphs, and confirms a conjecture of Espuny Díaz, Lichev, Mitsche, and Wesolek. Our proof is algorithmic and adapts to other families of graphs, in particular graphs with bounded genus or tree-width.

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The lattices $\mathbf{m} \times \mathbf{2}$ and $\mathbf{m} \times \mathbf{3}$ are not Schur positive

D.G.L. Wang* K. Zhang†

Abstract

We find two integer partitions λ and μ such that the s_λ-coefficient of the lattice $\mathbf{m} \times \mathbf{2}$ and the s_μ-coefficient of $\mathbf{m} \times \mathbf{3}$ are negative for $m \geq 8$. This result confirms a conjecture of Li, Qiu, Yang, and Zhang, and extends a counterexample to a comment of Stanley on the universal Schur positivity of distributive lattices. To the best of our knowledge, this is the first instance where Pieri's rules are used to prove the non-Schur positivity of a graph containing isolated vertices. The choice of the partitions λ and μ is inspired by the application of Pieri's rules. Moreover, using the notion of the strongly nice property—recently introduced and shown to be implied by Schur positivity—we further show that the lattice $\mathbf{m} \times \mathbf{3}$ is not strongly nice for $m \geq 44$.

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Relative Ordered Turán Densities

Dylan King*

Abstract

The Turán density of a graph F, $\pi(F)$, is the largest edge density in a host graph H which contains no copy of F. A well-known Theorem of Erdős, Stone, and Simonovits determines $\pi(F)$ as a function of the chromatic number $\chi(F)$. The relative Turán density of a graph F, $\rho(F)$, is the largest α so that every host graph H contains an F-free subgraph H' with $e(H') \geq \alpha \cdot e(H)$. By considering large cliques and utilizing a naïve random algorithm one quickly sees that, in fact, $\pi(F) = \rho(F)$.

In the setting of ordered graphs (where the vertex set is equipped with a total ordering $<$), Turán problems were studied systematically by Pach and Tardos, who gave a formula for the ordered Turán density $\pi_<(F)$ using the interval chromatic number $\chi_<(F)$. In 2025, Reiher, Rödl, Sales, and Schacht observed that, unlike the unordered setting, $\pi_<$ and $\rho_<$ no longer necessarily coincide. They initiated a formal study of $\rho_<$ and determined $\rho_<(P_\ell)$ where P_ℓ is the montone increasing path.

We resolve a question of Reiher, Rödl, Sales, and Schacht on the possible values of $\rho_<$ and exhibit several families of ordered matchings with $\rho_<(M) = 0$ (a result which was later improved to a complete characterization of $\rho_<(F) = 0$ by Illingworth, Ranganathan, Versteegen, and Williams).

Based on joint work with Bernard Lidický, Minghui Ouyang, Florian Pfender, Runze Wang, and Zimu Xiang completed at the 2025 Graduate Research Workshop in Combinatorics.

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Self-duality of pathwidth for polyhedral embeddings

Hikaru Yokoi*

Abstract

Given a graph G embedded on a closed surface, how similar are G and its (geometric) dual G^* in terms of width parameters? Research motivated by this question, so-called *self-duality of width parameters*, originated from the seminal work [J. Combin. Theory Ser. B 36 (1984)] in the *Graph Minors Project*. In this talk, we focus on self-duality of pathwidth and present a new bound for polyhedral embeddings, a generalization of 3-connected plane graphs. More precisely, we show that for a polyhedral embedding G on a closed surface with Euler characteristic χ, $\mathsf{pw}(G^*) \leq 3\,\mathsf{pw}(G)+c$, where $\mathsf{pw}(\cdot)$ denotes the pathwidth of a graph and c is a constant depending only on χ. This result improves the coefficient of $\mathsf{pw}(G)$ in the previously known bound by Fomin and Thilikos [J. Graph Theory 55 (2007)] and extends that of Amini, Huc, and Pérennes [SIAM J. Discrete Math. 23 (2009)] for plane graphs. Our approach is based on a new quantitative estimate which demonstrates the structural similarity between a polyhedral embedding and its dual.

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Winding number and circular coloring

Reza Naserasr [*†] Cyril Pujol[‡] Lujia Wang[§]

Abstract

In 1996, Youngs proved a surprising theorem that quadrangulations of the projective plane could never have chromatic number exactly 3. This sparked a lot of interest, and the result has been further developed in many directions over the past decades. In particular, it has been extended to the circular chromatic number, which is a real-valued lower bound on the chromatic number: The circular chromatic number of a quadrangulation of the projective plane cannot be in the interval $(2, 4)$. In this formulation, a gap for possible circular chromatic number is presented for projective planar graphs where all faces are $2k$-cycles.

In this work, we place these results into a unified framework based on the notion of winding number. This yields unified and simplified proofs of gaps in the circular chromatic number, that apply to graphs endowed with a distinguished set of even cycles. This generalizes the setting of graphs embedded on surfaces where every face is even.

We further establish an analogous gap phenomenon for faces of odd length, previously known only in the case of triangulations.

Finally, we extend this framework to signed graphs, where the analogy between vertex switching and $\mathbb{Z}_2$-actions provides a natural topological setting. We show that a circular coloring below a given threshold induces a continuous map from the signed graph, viewed as a topological object, to a circle. This provides a transparent connection between (signed) graph coloring and topology.

Using this approach, we show that the circular chromatic number of a signed graph embedded on the projective plane whose faces all have length k cannot be in the interval $(2, \frac{2k}{k-2})$ if the faces are positive, nor in the interval $(\frac{2k}{k-1}, \frac{2k-2}{k-2})$ if the faces are negative.

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Local arcs, locally repairable codes, large matchings

Ferdinand Ihringer*

Abstract

Motivated by the construction of optimal locally repairable codes, we introduce the new finite geometric concept of a *local arc* which is defined as a collection $\mathcal{S}$ of disjoint point sets S_i in $PG(2,q)$ such that $S_i \cup S_j$ is an arc for any $S_i, S_j \in \mathcal{S}$. In graph theoretic terms, this corresponds to a collection of vertex-disjoint complete bipartite graphs $K_{k,\binom{k}{2}}$ in the incidence graph of $PG(2,q)$. We will discuss upper and lower bounds. In particular, we adapt a recent breakthrough result by Hunter, Pohoata, Verstraete & Zhang, which shows that the incidence graph of $PG(2,q)$ has a matching of size at least $\Omega(q^{1.1667})$ to our setting. For $k = 4$, this implies the first superlinear construction of optimally locally repairable codes with minimum distance 6, locality 3, and disjoint repair groups.

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Frankl's diversity theorem for permutations

Eduard Inozemtsev* Andrey Kupavskii†

Abstract

For an intersecting family $\mathcal{F} \subset \binom{[n]}{k}$ we define its maximum degree $\Delta(\mathcal{F}) := \max_{x\in[n]} |\mathcal{F}[x]|$. Then *diversity* $\gamma(\mathcal{F})$ of the family $\mathcal{F}$ is $\gamma(\mathcal{F}) = |\mathcal{F}| - \Delta(\mathcal{F})$. That is, diversity is the number of sets not containing an element of maximum degree. Diversity is the (edit) distance to the closest star. In 1987, Frankl extended Hilton-Milner theorem and proved an influential stability result for the Erdős–Ko–Rado theorem, which in the version of Kupavskii-Zakharov upper bounds the size of $\mathcal{F} \subset \binom{[n]}{k}$ in terms of its diversity. We prove its analogue for the family of permutations on $\{1, \ldots, n\}$, provided $n \geq n_0$. This provides a similar extension of a Hilton–Milner type result for permutations proved by Ellis.

For $k = 2, \ldots, n-2$ consider a family $\mathcal{B}_k$ of all permutations that do not fix 1 and fix each of $k+1, \ldots, n$. Also consider a family $\mathcal{A}_k$ of all permutations that fix 1 and cross-intersect $\mathcal{B}_k$. It is easy to check that $|\mathcal{B}_k| = k! - (k-1)!$. If we set $\mathcal{E}_k = \mathcal{A}_k \sqcup \mathcal{B}_k$, then we have $\gamma(\mathcal{E}_k) = |\mathcal{B}_k|$. Also $|\mathcal{E}_{n-2}| = |\mathcal{E}_{n-3}|$ and recently Wang and Xiao proved that $\gamma(\mathcal{F}) \leq \gamma(\mathcal{E}_{n-2})$. We prove the following result.

Theorem. Let $n \geq n_0$. Take an intersecting family $\mathcal{F} \subset \Sigma_n$ of permutations. If for some integer $k \in [2, n-3]$ we have $\gamma(\mathcal{F}) \geq k! - (k-1)!$ then

$$|\mathcal{F}| \leq |\mathcal{E}_k|.$$

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Positivity Phenomena in Koornwinder Moments for the 2-ASEP

Younggwang Cho * Donghyun Kim † Jang Soo Kim ‡ Hojoon Lee § Jing Liu ¶
Minho Song‖

Abstract

The asymmetric exclusion process (ASEP) is a stochastic particle system on a one-dimensional lattice in which particles hop left and right with open boundaries. It is known that the partition function Z_n of the ASEP is closely related to the moments of the Askey–Wilson polynomials, which sit at the top of the hierarchy of classical orthogonal polynomials in one variable. On the other hand, the Askey–Wilson polynomials can be viewed as the one-variable case of the Koornwinder polynomials, also known as the Macdonald polynomials for the type BC root system. In 2018, Corteel and Williams showed that the partition function $Z_{n,k}$ of the two-species ASEP (2-ASEP) equals a special case of Koornwinder moments. Since Z_n is a polynomial with positive coefficients, this suggests a natural extension of the positivity of Z_n to Koornwinder moments. In this direction, Rains conjectured that the Koornwinder moments are also polynomials with positive coefficients, up to a normalization factor. We find an explicit formula for Koornwinder moments and prove the conjecture in two special cases. For $y = 1$ and $q = 0$, we introduce a lattice model and express Koornwinder moments as generating functions for families of nonintersecting lattice paths, so that positivity follows from the Lindström–Gessel–Viennot lemma. For $y = q = 1$, we evaluate Koornwinder moments explicitly, and also show that they can be interpreted combinatorially as the generating function for lecture hall tableaux.

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‖Yonsei University, Seoul, South Korea. Supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (No. 2022R1C1C2009025).

On degree powers in the degenerate Turán problem

Ping Hu* Ting Lan† Henry Liu‡

Abstract

Given a graph G with degree sequence $d_1, \ldots, d_n$ and a positive real number p, let $e_p(G) = \sum_{i=1}^{n} d_i^p$. For a fixed graph F, let $\mathrm{ex}_p(n, F)$ denote the maximum value of $e_p(G)$ over all F-free graphs G on n vertices. In 2000, Caro and Yuster introduced the following Turán-type problem: For a positive integer p, determine $\mathrm{ex}_p(n, F)$, and characterize the extremal graphs G on n vertices that attain $\mathrm{ex}_p(n, F)$. For the case when F is a bipartite graph, the function $\mathrm{ex}_p(n, F)$ has previously been studied by various authors, including when F is a path, a star, and a complete bipartite graph (Caro and Yuster); an even cycle (Nikiforov); a linear forest and a star forest (Lan, Liu, Qin and Shi); and a broom (Gerbner). Recently, Gao, Liu, Ma and Pikhurko proved that $\mathrm{ex}_p(n, F) = (\tau(F) - 1 + o(1))n^p$ for real $p > \frac{1}{1-\alpha}$, where F is a bipartite graph with classical Turán number $\mathrm{ex}(n, F) = O(n^{1+\alpha})$ for some $\alpha \in [0, 1)$, and $\tau(F)$ is the minimum size of an independent covering set of F (in fact they proved a version of this result for hypergraphs). Based on their method, we obtain a stability result for $\mathrm{ex}_p(n, F)$, and prove that all extremal graphs must contain the complete bipartite graph $K_{\tau(F)-1, n-\tau(F)+1}$ when n is sufficiently large. Our results can be used to deduce all previously known results about $\mathrm{ex}_p(n, F)$ when F is a bipartite graph and n is large, as well as many new exact results, including when F is an even cycle and a complete bipartite graph.

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A characterization of graphs with no $K_{3,4}$ minor

On-Hei Solomon Lo*

Abstract

Wagner established a characterization of graphs with no $K_{3,3}$ minor in 1937. Following this line of research, we provide a characterization of graphs with no $K_{3,4}$ minor. As a consequence, we prove that every 4-connected non-planar graph with at least seven vertices and minimum degree at least five contains both $K_{3,4}$ and K_6^- as minors, thereby proving a conjecture of Kawarabayashi and Maharry in a strong form.

*Tongji University, Shanghai, China. Supported by the Fundamental Research Funds for the Central Universities.

Indicated List Coloring Game On Graphs

Yangyan Gu* Yiting Jiang† Huan Zhou‡ Jialu Zhu§ Xuding Zhu¶

Abstract

Given a graph G and a list assignment L for G, the indicated L-coloring game on G is played by two players: Ann and Ben. In each round, Ann chooses an uncolored vertex v, and Ben colors v with a color from $L(v)$ that is not used by its colored neighbors. If all vertices are colored, then Ann wins the game. Otherwise after a finite number of rounds, there remains an uncolored vertex v such that all colors in $L(v)$ have been used by its colored neighbors, in which case Ben wins. We say G is indicated L-colorable if Ann has a winning strategy for the indicated L-coloring game on G. For a mapping $g : V(G) \to \mathbb{N}$, we say G is indicated g-choosable if G is indicated L-colorable for every list assignment L with $|L(v)| \geq g(v)$ for each vertex v, and G is indicated degree-choosable if G is indicated g-choosable for $g(v) = d_G(v)$ (the degree of v). In this talk we show that a graph G is not indicated degree-choosable if and only if G is an expanded Gallai-tree - a graph whose maximal connected induced subgraphs with no clique-cut are complete graphs or blow-ups of odd cycles, along with a minor technical condition. This leads to a linear-time algorithm that determines if a graph is indicated degree-choosable.

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The chromatic number of finite projective spaces

Anurag Bishnoi* Wouter Cames van Batenburg † Ananthakrishnan Ravi‡

Abstract

The chromatic number of the finite projective space $\mathrm{PG}(n-1,q)$, denoted $\chi_q(n)$, is the minimum number of colors needed to color its points so that no line is monochromatic. We establish the following recursive bound that shows subadditivity of the chromatic number: $\chi_q(n) \leq \chi_q(d) + \chi_q(n-d)$ for all $1 \leq d < n$, and use it to prove new upper bounds on $\chi_q(n)$ for all q. For $q = 2$, we further refine this recursion and prove that

$$\chi_2(n) \leq \lfloor 2n/3 \rfloor + 1$$

for all $n \geq 2$, and that this bound is tight for all $n \leq 7$. In particular, this recovers all previously known cases for $n \leq 6$ and resolves the first open case $n = 7$. On the lower-bound side, using a connection with multicolor Ramsey numbers for triangles, we note that

$$\chi_2(n) \geq (1 - o(1)) \frac{n}{\log n}.$$

We also consider $\chi_q(t;n)$, the minimum number of colors needed to color the points of $\mathrm{PG}(n-1,q)$ with no monochromatic $(t-1)$-dimensional subspace, and establish an equivalence between $\chi_q(t;n)$ and the multicolor vector-space Ramsey numbers $R_q(t;k)$. Using this equivalence together with new upper bounds on $\chi_q(t;n)$, we improve, for every fixed t, the best known lower bounds on $R_q(t;k)$ from $\Omega_q(\log k)$ to $\Omega(k)$.

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A complete t-intersection theorem for families of spanning trees

Elizaveta Iarovikova[*] Andrey Kupavskii [†]

Abstract

A family of sets $\mathcal{F}$ is *t-intersecting* if for any $A, B \in \mathcal{F}$ we have $|A \cap B| \geq t$. The study of t-intersecting families originated in the seminal paper of Erdős, Ko and Rado, who studied maximal t-intersecting families in $2^{[n]}$ and $\binom{[n]}{k}$. Over the last 65 years, the Erdős–Ko–Rado type theorems were obtained for a wide range of structures.

However, complete t-intersection theorems (i.e. results valid for all relevant t) to the best of our knowledge are obtained only in three settings: for $2^{[n]}$ (Katona), for $\binom{[n]}{k}$ (Ahlswede and Khachatrian), and for k-dimensional subspaces of an n-dimensional vector space over $\mathbb{F}_q$ (Frankl and Wilson). We establish one more result of this kind: a complete t-intersection theorem for families of spanning trees.

Let $\mathcal{T}_n$ denote the set of labelled spanning trees in a complete graph on n vertices. We say that a family $\mathcal{F} \subset \mathcal{T}_n$ is *t-intersecting* if for all $A, B \in \mathcal{F}$ the trees A and B share at least t edges. We show that the following theorem holds for all sufficiently large n.

Theorem 1. *For all $2 \leq t \leq n-1$, the size of a t-intersecting family $\mathcal{F} \subset \mathcal{T}_n$ is at most $c_{n,t}n^{n-2-t}$, where $c_{n,t}$ is the maximum product of $n-t$ positive integers with sum n. Moreover, equality holds if and only if $\mathcal{F}$ is the family of all spanning trees containing a fixed forest F with t edges and connected components of almost equal sizes.*

Previously, this problem was solved for $1 \leq t \leq Cn/\log_2 n$ by Frankl, Hurlbert, Ihringer, Kupavskii, Meagher and Pantangi. Independently and contemporaneously to our studies, Saengrungkongka proved the same theorem for $1 \leq t \leq \varepsilon n$.

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Extremal density for subdivisions with length or sparsity constraints

Fan Yang*

Abstract

Given a graph H, a balanced subdivision of H is obtained by replacing all edges of H with internally disjoint paths of the same length. In this paper, we prove that for any graph H, a linear-in-$e(H)$ bound on average degree guarantees a balanced H-subdivision. This strengthens an old result of Bollobás and Thomason, and resolves a question of Gil-Fernández, Hyde, Liu, Pikhurko and Wu.

We observe that this linear bound on average degree is best possible whenever H is logarithmically dense. We further show that this logarithmic density is the critical threshold: for many graphs H below this density, its subdivisions are forcible by a sublinear-in-$e(H)$ bound on average degree. We provide such examples by proving that the subdivisions of any almost bipartite graph H with sublogarithmic density are forcible by a sublinear-in-$e(H)$ bound on average degree, provided that H satisfies some additional separability condition.

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From Halin's Edge Removability to Matching Removability in k-connected graphs

Shinya Fujita[*] Hengzhe Li[†] Yaping Mao[‡] Mingming Zhou[§]

Abstract

We discuss the matching-removability under the degree/connectivity regime of Halin's theorem, which asserts that every k-connected graph G with minimum degree $\delta(G) \geq k+1$ contains an edge e such that $G-e$ remains k-connected. For $k, \ell \geq 1$, an ℓ-matching is a matching of size ℓ. A matching M in a k-connected graph G is *k-removable* if $G-M$ remains k-connected. We improve Halin's result by proving that every k-connected graph G with $\delta(G) \geq k+1$ contains a k-removable 2-matching, except when $k=1$ and G is a cycle. Moreover, for small k we obtain stronger bounds on the size of removable matchings. Roughly, we discuss the following three cases: (i) $k=1$: a 1-removable $\min\{\lfloor n/2 \rfloor, \delta(G)\}$-matching; (ii) $k=2$: a 2-removable $\lceil(\delta(G)+1)/2\rceil$-matching, with a unique tight exception when $\delta(G)$ is even and $G \cong K_{\delta(G)+1}$; and (iii) $k=3$: for $\delta(G) \geq 5$, a 3-removable $\lceil(\delta(G)+1)/2\rceil$-matching.

We also show that for every $n \geq 2\delta$, there exists a k-connected n-vertex graph G with minimum degree δ that does not contain a k-removable matching of size at least $\delta(G)+1$. Some more constructions concerning the sharpness on the size of removable matchings will be presented.

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Cubic graphs with game chromatic number 3

Naoki Matsumoto*

Abstract

The game chromatic number of a graph is a variation of the classical chromatic number defined via a two-player coloring game, where players alternately color a vertex subject to the usual proper coloring rule. The first player aims to color all vertices using the given number of colors, while the second player aims to prevent this. The game chromatic number of a graph, denoted by $\chi_g(G)$, is the minimum number of colors for which the first player has a winning strategy, that is, a strategy that guarantees that all vertices are eventually properly colored.

In this talk, we focus on cubic graphs, that is, graphs in which every vertex has degree three. It is easy to see that for every simple cubic graph G with at least three vertices, $3 \leq \chi_g(G) \leq 4$. Frieze et al. [SIAM J. Discrete Math. 27 (2013)] proved that for a random simple cubic graph G, $\chi_g(G) = 4$ with high probability. This naturally leads to the following question: which simple cubic graphs satisfy $\chi_g(G) = 3$?

In this talk, we present several observations and results toward this problem, along with related questions and future directions.

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Designs of Perfect Matchings

John Bamberg* Lukas Klawuhn†

March 29, 2026

Abstract

Perfect matchings, also called 1-*factors*, are classical objects with abundant connections to many areas of combinatorics. It is well-known that the complete graph K_{2n} on $2n$ vertices can always be decomposed into perfect matchings, called a 1-factorisation. In such a decomposition, every edge of K_{2n} appears in exactly 1 perfect matching.

This was generalised by Jungnickel and Vanstone to *hyperfactorisations*. These are sets of perfect matchings such that every pair of disjoint edges of K_{2n} appears in a constant number of perfect matchings. Hyperfactorisations are examples of Cameron's partition systems and were rediscovered by Stinson who called them *hyperresolutions*. We generalise all these ideas to λ-factorisations of K_{2n} and characterise them algebraically as Delsarte designs in an association scheme obtained from the Gelfand pair $(S_{2n}, S_2 \wr S_n)$. We use this characterisation to derive divisibility conditions and non-existence results. This unifies and extends results by Cameron (1976). Furthermore, we explore a connection to finite geometry using (abstract) hyperovals, giving rise to explicit constructions of λ-factorisations. We also give a short census for small values of n, showing which λ- and hyperfactorisations are known to exist and what the smallest open cases are. Our results improve on the best previously known structures.

Our methods include working in the group algebra $\mathbb{C}[S_{2n}]$ and using the representation theory of S_{2n}.

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Intersection theorems in spread domains

Andrey Kupavskii*

Abstract

In this talk, we will discuss some of the recent extremal set theoretic results in spread domains, including sets, permutations and vector spaces. This includes the results that discern a low-uniformity (or low-dimensional) structure in any large intersecting and t-intersecting family, as well as extremal and stability results. In particular, we will discuss results for vector spaces in the spirit of Frankl's degree-diversity theorem and recent progress on the problem of Deza and Frankl on t-intersecting families of permutations.

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Contributed Talk Abstracts

Wednesday, August 26

Lascoux's series, parking functions and noncrossing partitions

Alice L.L. Gao* Xin-Bei Liu† Arthur L.B. Yang‡ James J.Y. Zhao§

Abstract

Given a partition $\lambda = (\lambda_1, \ldots, \lambda_n)$ and a permutation $\sigma \in \mathfrak{S}_n$, Lascoux studied the generating series of Demazure characters

$$1 + \kappa_{\lambda,\sigma}(\mathbf{x})t + \kappa_{2\lambda,\sigma}(\mathbf{x})t^2 + \kappa_{3\lambda,\sigma}(\mathbf{x})t^3 + \cdots,$$

whose specialization at $\mathbf{x} = \mathbf{1}$ encodes the dimensions of the spaces of global sections of the line bundle $L^{k\lambda}$ over the Schubert subvariety Ω_σ for all integers $k \geq 0$. Given λ and σ, let

$$F_{\lambda,\sigma}(t) = \sum_{k\geq 0} \kappa_{k\lambda,\sigma}(\mathbf{1})t^k,$$

when written in lowest terms, its numerators is denoted by $\mathcal{E}_{\lambda,\sigma}(t)$. Lascoux proposed an open problem that to give a satisfactory expression of $\mathcal{E}_{\lambda,\sigma}(t)$. In this paper we obtain a combinatorial interpretation of $\mathcal{E}_{\delta_n,\sigma}(t)$ for the staircase partition δ_n and the permutation $\sigma = [2, 3, \ldots, n, 1]$ and its inverse in terms of the descent statistic of parking functions of length $n - 1$. Based on this progress on Lascoux's open problem, we find that the polynomial $\mathcal{E}_{\delta_n,\sigma}(t)$ for this special case coincides with the h-polynomial $h(\Delta(\mathrm{NC}_W), t)$ of the order complex of the noncrossing partition lattice associated to the irreducible Coxeter group W of type A_{n-1}. This coincidence inspires us to give an operator approach to $h(\Delta(\mathrm{NC}_W), t)$ for any finite Coxeter group W. As applications, we explore some analytic properties of $h(\Delta(\mathrm{NC}_W), t)$ and completely solve an open problem on $h(\Delta(\mathrm{NC}_W), t)$ which was proposed by Athanasiadis, Douvropoulos, and Kalampogia-Evangelinou.

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Monochromatic matchings in hypergraphs

Zhifei Yan*

Abstract

The chromatic number of Kneser hypergraphs was determined by Alon, Frankl, and Lovász (AFL) in 1986, resolving a conjecture of Erdős. Equivalently, their result asserts that every r-edge-colouring of the complete t-uniform hypergraph on n vertices contains a monochromatic matching of size

$$\left\lfloor \frac{n+t-1}{r+t-1} \right\rfloor .$$

While the AFL theorem has had a profound influence over the past several decades, all known proofs and subsequent developments rely on a non-constructive topological framework. Finding an elementary proof has been regarded as a central open problem.

In this paper, we introduce a fundamentally different, topology-free framework for hypergraph colouring problems via monochromatic matchings. Our approach is constructive and elementary, relying on weak hypergraph regularity and linear programming duality. The core of our method is a structural reduction: we show that any edge-colouring of a pseudo-random hypergraph can be structurally reduced to a specific colouring pattern, preserving the maximal monochromatic matching size up to an $o(n)$ error.

This framework is robust and admits several applications beyond the classical setting:

- We obtain the first constructive and elementary proof of the asymptotic AFL theorem: for $t = O(1)$ and $q = \Omega(n)$, we determine $\chi\big(\mathrm{KG}^q(n,t)\big)$ up to an additive error of 1.
- We provide a new proof of the transference AFL theorem for random hypergraphs, recently established by Gishboliner, Glock, Michaeli and Sgueglia.
- For the stable Kneser hypergraph $\mathrm{KG}^q(n,t)_{s\text{-stab}}$, we confirm Meunier's conjecture up to an additive error of 1 in the regime where $t = O(1)$, $q = \Omega(n)$, and $s \ll q$.

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p-competitively orientable graphs

Taehee Hong*

Abstract

A graph G is said to be p-competitively orientable if there exists a orientation of G such that any two vertices have p common out-neighbors. In [Choi et al. Competitively orientable complete multipartite graphs, Discrete Mathematics, 345(9):112950, 2022], Choi et al. completely characterized complete graphs and complete multipartite graphs each of which is 1-competitively orientable. We aim to extend their results to the general case of p. We first observe the relationship between p-competitive orientations for certain complete graphs and Hadamard matrices. The main result of this paper is the derivation of a bound on the order of a complete graph that admits a p-competitive orientation. Furthermore, based on observation that no complete bipartite graph is p-competitively orientable for any positive integer p, we derive necessary conditions and sufficient conditions for complete tripartite graphs to admit a p-competitive orientation. This is a joint work with Myungho Choi and Boram Park.

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Edge-coloring 4- and 5-regular projective planar graphs with no Petersen-minor

Arnott Kidner* Eckhard Steffen† Weiqiang Yu‡

Abstract

An r-regular graph is an r-graph, if every odd set of vertices is connected to its complement by at least r edges. A snark is a cyclically 4-edge connected cubic graph with girth at least 5, that is not 3-edge-colorable. The Four Color Theorem states that there is no planar snark. In [arXiv:2405.16586 (2024)], it is shown that the Petersen graph is the only projective planar snark. We study projective planar r-graphs for $r \in \{4, 5\}$ and show that they are r-edge-colorable if they have no Petersen-minor.

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Curvilinear Tilings and Hurwitz Problem

Haofang Sun* Min Yan†

March 19

Abstract

The classical Hurwitz realisation problem was largely solved by Edmond, Kulkarni, Stong in 1980's. Some of their work was done by relating to tilings of surfaces. In this talk, we revisit the problem from the fresh angle of edge-to-edge tilings of surfaces by congruent curvilinear polygons, where by curvilinear we mean the edges are not necessarily straight. We concentrate on the most extreme curvilinear case, where the polygon is equivalent to a planar diagram D.

Let $V_1, V_2, \ldots, V_k$ be vertices in the surface S_D determined by the planar diagram D. Then the vertices in a tiling by D are $V_1^{k_1}, V_2^{k_2}, \ldots, V_p^{k_p}$. The tiling gives a branched cover over S_D, with k_i as the branching data. Our first result is the necessary and sufficient condition for a branched data to come from a tiling.

The classification of tilings of all surfaces by the given planar diagram D is the same as finding "minimal tilings", which are tilings that do not cover other tilings. Using the first result, we get the second main result: For most D, satisfying $\chi(S_D) \leq 0$, the number of minimal tilings with the given vertices $V_1^{k_1}, V_2^{k_2}, \ldots, V_p^{k_p}$ is infinite. This shows the classification problem can be hard to solve for all surfaces. We also discuss minimal surfaces for surfaces of fixed genus.

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Algorithmic Applications of Reduced Component Max-Leaf

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March 28, 2026

Abstract

Reduced parameters [BKW, JCTB '26; BKRT, SODA '22] are defined via contraction sequences. Based on this framework, we introduce the reduced component max-leaf, where component max-leaf is the maximum number of leaves in any spanning tree of any connected component. Reduced component max-leaf lies strictly between clique-width and reduced bandwidth.

In this talk, we focus on its algorithmic applications. We consider the σ-Neighborhood problem: given a finite set $\sigma \subseteq \mathbb{N} \cup \{0\}$, the task is to find a maximum-size vertex set $S \subseteq V(G)$ such that $|N_G(v) \cap S| \in \sigma$ for every $v \in S$. We show that σ-Neighborhood problem can be solved in polynomial time on graph classes of bounded reduced component max-leaf, provided a certifying contraction sequence is given. This problem naturally captures Maximum Independent Set, Maximum Induced Matching and Maximum Induced d-Regular Subgraph. Since the reduced component max-leaf is preserved under graph complementation, Maximum Clique is also solvable in polynomial time on these classes. Furthermore, we also design a polynomial-time algorithm for Induced Disjoint Paths, which is not captured by the σ-Neighborhood problem. These results extend known tractability results for classes of bounded clique-width.

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The K_0-Ring of Spanning Line Configurations, Fubini Words, and Rectangular Pipe Dreams

Michael Ruofan Zeng *

Abstract

The generalized coinvariant algebra $R_{n,k}$ of Haglund–Rhoades–Shimozono is a natural generalization of the classical coinvariant algebra R_n. Pawlowski–Rhoades' variety of spanning line configurations $X_{n,k}$ is the geometric model for $R_{n,k}$ and has deep connections to the combinatorics of Fubini words. The main result of this talk is a canonical ring isomorphism $K_0(X_{n,k}) \cong R_{n,k}$, giving the K-theoretic analogue of the Pawlowski–Rhoades presentation of the cohomology $X_{n,k}$ and extending the classical picture for the flag variety. The key new input is a localization theorem in K-theory for complements of unions of cell closures in smooth cellular varieties. Using the Fulton–Lascoux degeneracy locus formula, we recover $R_{n,k}$ as an explicit presentation of $K_0(X_{n,k})$ in terms of tautological line bundles. We identify the fundamental classes of Pawlowski–Rhoades varieties with Grothendieck polynomials of Fubini words. Motivated by this geometric identification, we introduce classical and bumpless *rectangular pipe dreams* for any word $w \in [k]^n$, extending the usual permutation-based models to Fubini words. These models show that Schubert polynomials and Grothendieck polynomials of words arise as weight generating functions for rectangular pipe dreams. Altogether, the results give a generalized version of Schubert calculus under the setting of $X_{n,k}$.

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Berge tight cycles of all lengths in hypergraphs

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Abstract

Given a set R of positive integers, an R-graph $H = (V, E)$ is a hypergraph where the cardinality of each hyperedge belongs to R. If $R = \{r\}$, we sometimes refer to the hypergraph as an r-graph rather than an R-graph. For a set $S \subseteq V$, let $d_H(S)$ denote the number of hyperedges of H containing S. Given a nonnegative integer s, the minimum s-degree $\delta_s(H)$ is the minimum of $d_H(S)$ over all s-vertex subsets S of V. Let r and t be positive integers with $r < t$. We denote by C_t^r the t-vertex tight cycle, which is an r-graph admitting a cyclic ordering of its vertices where every r consecutive vertices form a hyperedge. In particular, C_t^2 is equivalent to the classical cycle C_t in 2-graph. For hypergraphs F and H, we say that H is a Berge-F if there exist an injection $f\colon V(F) \to V(H)$ and a bijection $g\colon E(F) \to E(H)$ such that $\{f(v) : v \in e\} \subseteq g(e)$ for all $e \in E(F)$.

Lu and Wang [Discrete Math. 344 (2021), 112462] proved that every [3]-graph H on $n \geq 6$ vertices with $\delta_2(H) \geq 1$ contains a Berge-C_t for all $3 \leq t \leq n$. In this manuscript, we prove that for any positive integer r and any set $R \subseteq [k]$ with $k \geq 2$, there exists an integer $n_0 = n_0(k, r)$ such that every R-graph H on $n \geq n_0$ vertices with $\delta_r(H) \geq 1$ contains a Berge-C_t^r for all $r+1 \leq t \leq n$. In particular, when $k = 4$ and $r = 3$, we show that every [4]-graph H on $n \geq 9$ vertices with $\delta_3(H) \geq 1$ contains a Berge-C_t^3 for all $4 \leq t \leq n$. We also characterize all the counterexamples when $4 \leq n \leq 8$.

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An Improved Bound of Stein's Conjecture for Antidirected Paths

Yaobin Chen* Yulai Ma† Chaoliang Tang‡ Junchi Zhang§ Wentao Zhang¶

Abstract

Stein's conjecture [Discrete Mathematics and Applications (2020)] states that every oriented graph G with minimum semidegree $\delta^{\pm}(G) > k/2$ contains every orientation of a path of length k. This conjecture is sharp considering a cyclically oriented blow-up of $k/2$ vertices per part. We prove that for any integer $k \geq 4$, every oriented graph with minimum semidegree greater than $\frac{k}{2} + k^{1/3}$ contains an antidirected path of length k. This improves the previous bound $\frac{1}{2}(k - 1 + \sqrt{k-3})$ due to Grzesik and Skrzypczyk [arXiv:2506.11866 (2025)] and narrows the gap toward Stein's conjecture.

Theorem 1. *For any integer $k \geq 4$, every oriented graph G with $\delta^{\pm}(G) > \frac{k}{2} + k^{1/3}$ contains an antidirected path of length k.*

Our proof builds on the structural framework introduced in [arXiv:2506.11866 (2025)], particularly a lemma that allows us to assume the existence of a large set F of in-neighbors of v_1 outside the antipath. Our key contribution is to treat vertices of F as *switchers* to construct longer paths. A further direction is to investigate the existence of a longest antipath where **both** ends have atleast two neighbors outside the path. We believe the answer of this question, if ture, together with the technique we use in this paper, could eventually close the gap.

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Degree-truncated choice number of planar graphs

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Abstract

Assume G is a graph and k is a positive integer. Let $f : V(G) \to \mathbb{N}$ be defined as $f(v) = \min\{k, d_G(v)\}$. If G is f-choosable, then we say G is degree-truncated k-choosable. The degree-truncated choice number of G is $\mathrm{ch}^{\mathrm{d}}(G) = \min\{k : G \text{ is degree-truncated } k\text{-choosable}\}$. For a family $\mathcal{G}$ of graphs, $\mathrm{ch}^{\mathrm{d}}(\mathcal{G}) = \max\{\mathrm{ch}^{\mathrm{d}}(G) : G \in \mathcal{G}\}$. Let $\mathcal{P}$ denote the family of 3-connected non-complete planar graphs. Richter asked in 2008 whether $ch^{\mathrm{d}}(\mathcal{P}) \leq 6$. In 2025, Zhou, Zhu and Zhu answered this question in negative and proved that $8 \leq ch^{\mathrm{d}}(\mathcal{P}) \leq 16$ [Degree-truncated choosability of graphs, J. Combin. Theory Ser. B, 175 (2025), 171–186]. This result was improved by Jiang, Xu, Xu, and Zhu, who proved that $9 \leq ch^{\mathrm{d}}(\mathcal{P}) \leq 12$. In this paper, we further improve the result and prove that $10 \leq \mathrm{ch}^{\mathrm{d}}(\mathcal{P}) \leq 11$. We conjecture that $\mathrm{ch}^{\mathrm{d}}(\mathcal{P}) = 10$, and we confirm this conjecture for those planar graphs $G \in \mathcal{P}$ for which the subgraph induced by vertices of degree at least 11 is 4-choosable.

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Spanning triangulations in plane X-mosaics

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Abstract

A graph is 1-*planar* if it can be drawn on the plane so that each edge crosses at most one other edge. It is known that every 1-planar graph G with at least three vertices satisfies $|E(G)| \leq 4|V(G)|-8$. A 1-planar graph G is *optimal* if it satisfies $|E(G)| = 4|V(G)|-8$. An edge in a 1-planar graph is *crossing* if it crosses another edge, and *non-crossing* otherwise. It was shown that every optimal 1-planar graph contains a 4-connected triangulation as a spanning subgraph, and that, for an optimal 1-planar graph G, the spanning subgraph induced by all non-crossing edges of G is a 3-connected quadrangulation. For optimal 1-planar graphs, several results have been obtained by focusing on the existence of spanning subgraphs as described above. In this talk, we focus on the property of containing a "good" spanning subgraph, and introduce a more general class that includes the class of optimal 1-planar graphs. A 1-planar graph G is an *X-mosaic* if it contains a triangulation as a spanning subgraph. Note that every plane triangulation is an X-mosaic. In particular, we discuss the connectivity of X-mosaics and their spanning triangulations.

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Regular Graphs with given Automorphism Groups

Reymond Akpanya*

Abstract

Given a finite group G and an arbitrary integer $d \geq 3$, G. Sabidussi proposed a construction of a d-regular graph whose automorphism group is isomorphic to G, see [Canad. J. Math. 9 (1957)]. His approach relies on the notion of graph products together with a cubic graph construction introduced by R. Frucht in 1949, see [Math. Z. 72 (1959/60)] and [J. Algebr. Comb. 62 (2025); Canad. J. Math. 1 (1949)]. In this talk, we revisit Sabidussi's construction of d-regular graphs with prescribed automorphism groups. We identify an oversight in the original argument and present an alternative construction that strengthens Sabidussi's result. In addition, we show that for every $d \geq 3$ and every finite group G there exists a d-regular graph Γ with a strong embedding β such that $\mathrm{Aut}(\Gamma) \cong \mathrm{Aut}(\beta(\Gamma)) \cong G$.

This is joint work with Meike Weiß (RWTH Aachen) and Tom Goertzen (USyd).

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Deograms and their links to rational Catalan objects

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Abstract

Arising from Deodhar's work, two families of combinatorial objects called $\mathbf{f}_{k,n}$-Deograms and $\mathbf{c}_{k,n}$-Deograms were introduced in different mathematical contexts. Although it has been shown that both families are enumerated by the rational Catalan numbers, no conceptual explanation for this coincidence was previously known. In fact, finding a direct bijection between them was posed as an open problem. In this talk, we briefly discuss the origins of these two kinds of Deograms and present a bijective explanation of their equinumerosity. We further specialize to the Fuss–Catalan case and describe explicitly a bijection between Deograms and rational Dyck paths of Fuss–Catalan type. We also discuss several natural problems that remain for future study. *This is joint work with Byung-Hak Hwang and Jihyeug Jang.*

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An Erdős Matching Conjecture for Vector Spaces

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Abstract

For an n-dimensional vector space V over $\mathbb{F}_q$, let $\begin{bmatrix} V \\ k \end{bmatrix}$ denote the family of all k-dimensional subspaces of V and let $\left|\begin{bmatrix} V \\ k \end{bmatrix}\right| := \begin{bmatrix} n \\ k \end{bmatrix}_q$. We say that $F_1, \ldots, F_s \in \begin{bmatrix} V \\ k \end{bmatrix}$ are mutually independent if $\dim(F_1 + \cdots + F_s) = sk$, and let $m_q(n, k, s)$ be the maximum size of a family $\mathcal{F} \subseteq \begin{bmatrix} V \\ k \end{bmatrix}$ containing no $s+1$ mutually independent members. We formulate the following Erdős Matching Conjecture for vector spaces: for all $n \geq (s+1)k$,

$$m_q(n, k, s) = \max \left\{ \begin{bmatrix} (s+1)k - 1 \\ k \end{bmatrix}_q, \begin{bmatrix} n \\ k \end{bmatrix}_q - q^{ks} \begin{bmatrix} n-s \\ k \end{bmatrix}_q \right\}.$$

When $s = 1$, this reduces to the Erdős–Ko–Rado theorem for vector spaces, proved by Frankl and Wilson in 1987.

We verify the vector-space EMC for a broad range of parameters, namely $k = 2$, $n = (s+1)k$, and $n \geq (2s+1)k - s + 1$. The case $k = 2$ may be viewed as a vector-space analogue of the Erdős–Gallai theorem, while the case $n = (s+1)k$ follows from a universal bound obtained via a high-dimensional Hoffman inequality.

A notable feature of this problem is its close connection with vector-space cover-free families, extending the classical notion introduced by Erdős, Frankl, and Füredi in 1985. We say that a family $\mathcal{F} \subseteq \begin{bmatrix} V \\ r \end{bmatrix}$ is t-cover-free if there do not exist $F, F_1, \ldots, F_t \in \mathcal{F}$ such that $F \subseteq \sum_{i=1}^{t}(F \cap F_i)$. We establish a new connection between the vector-space EMC and vector-space cover-free families, and determine the maximum size of a t-cover-free family in $\begin{bmatrix} V \\ r \end{bmatrix}$ up to a lower-order term for all fixed q, t, r as $n \to \infty$. This extends a recent result of Shang and Zhou in the case $t = 2$.

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Locally Irregular Graph Covering

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Abstract

A locally irregular graph is a graph where each pair of adjacent vertices has differing degrees. In 2015, Baudon, Bensmail, Przybyło, and Woźniak conjectured that if the edges of a graph G can be partitioned into locally irregular graphs, then three locally irregular graphs suffice. This was disproven by Sedlar and Škrekovski who gave an example of a graph needing four locally irregular graphs to partition its edges. The best known upper bound for the minimum number of locally irregular graphs needed to partition the edges of G is 220, due to Lužar, Przybyło, and Soták.

In this work, we introduce a related setting where the edges of the locally irregular subgraphs are allowed to overlap. In particular, we define the *locally irregular cover number*, $lic(G)$, as the minimum number of locally irregular subgraphs of G such that every edge of G is contained in at least one of them. Unlike in the original setting, this value exists for any $G \neq K_2$.

With the exceptions of K_2 and K_4, we show that $lic(G) = 2$ for any tree, complete graph, or complete multipartite graph. For bipartite graphs, we show that $lic(G) \leq 6$, improving the bound implied by earlier work in the partition setting. On the other hand, we also construct an infinite family of graphs, containing arbitrarily many cycles of any odd size, for which $lic(G) \geq 3$.

Notably, $lic(G) = 2$ for the only known graph which requires four locally irregular graphs to be partitioned without overlap. With this in mind, we conjecture that $lic(G)$ is always at most 3 and that it is at most 2 when G is bipartite.

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Odd edge colorings of graphs with high connectivity

Mikio Kano[*] Shun-ichi Maezawa[†] Kenta Ozeki[‡]

Abstract

An *odd subgraph* of a graph is a subgraph in which every vertex has odd degree. A graph G is said to be *odd k-edge-colorable* if there exists an edge-coloring $E(G) \to \{1, 2, \dots, k\}$ such that each nonempty color class induces an odd subgraph of G. The *odd chromatic index* of G, denoted by $\chi'_o(G)$, is the minimum k for which G is odd k-edge-colorable.

Pyber [Colloq. Math. Soc. János Bolyai 60 (1991)] proved that every connected graph G of even order satisfies $\chi'_o(G) \leq 3$. He also observed that the wheel W_4 with four spokes satisfies $\chi'_o(W_4) = 4$, and asked whether there exist infinitely many connected graphs G with $\chi'_o(G) = 4$. Mátrai [J. Graph Theory 53 (2006)] answered this question affirmatively, but all of his examples contain a cut-vertex. Later, Atanasov, Petruševski, and Škrekovski [Ars Math. Contemp. 10 (2016)] constructed infinitely many 2-connected graphs G with $\chi'_o(G) = 4$. However, apart from W_4, no examples with higher connectivity have been found.

Motivated by this situation, we prove that every 4-connected simple graph of odd order is odd 3-edge-colorable, and show that there are infinitely many 3-connected graphs G with $\chi'_o(G) = 4$.

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Helly-type Theorems for Multiple Piercing

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Abstract

Let Π^m denote the property that a family of sets can be pierced by m points, and let Π^m_k denote the property that any subfamily of size at most k can be pierced by m points. A family has m-Helly number at most k if Π^m_k implies Π^m, and it has m-Helly property if it has a finite m-Helly number.

Helly's theorem says that the family of compact convex sets in $\mathbb{R}^d$ has 1-Helly number $d+1$. It is natural to ask whether compact convex sets in $\mathbb{R}^d$ still has the m-Helly property for $m \geq 2$.

Even in the plane, however, the family of compact convex sets fails to have the 2-Helly property. This raises a weaker question: does Π^2_k implies Π^3 for sufficiently large k? We show that Π^2_k implies Π^5 for compact convex sets in the plane when k is sufficiently large. The proof adapts the Alon-Kleitman method for the (p, q)-theorem, combining Kalai's upper bound theorem, linear programming duality, and bounds on weak ε-net. We also show that bounded-degree affine hypersurface has the m-Helly property for every m, and we obtain colorful generalizations of bounded m-Helly property for certain special families.

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Basis Number of Graphs Excluding Minors

Colin Geniet* Ugo Giocanti†

Abstract

The *cycle space* of a graph G is the $\mathbb{F}_2$-vector-space of all eulerian subgraphs of G, with symmetric difference as sum. In other words, it is the subspace generated by all cycles of G. A *cycle basis* is a basis of the cycle space. For instance, in planar graphs, the set of facial cycles form a cycle basis $\mathcal{B}$; in this specific case, note that each edge is used by at most two cycles of $\mathcal{B}$. Mac Lane ('37) proved that this is a characterisation: a graph is planar if and only if it has a cycle basis using each edge at most twice. Generalising this, Schmeichel ('81) defined the *basis number* of G as the smallest k such that G has a cycle basis using each edge at most k times.

Graphs embedded on a surface of bounded genus are known to have bounded basis number; this was recently improved to a poly-logarithmic bound by Lehner and Miraftab ('24). Generalising this, we prove that graphs excluding a fixed minor H have bounded basis number. Our proof uses the Graph Minor Structure Theorem, and the main difficulty lies in understanding how basis number behaves in tree-decompositions. In particular, we prove that graphs of treewidth k have basis number bounded by some function of k. We handle tree-decompositions using the proof framework developed Bojańczyk and Pilipczuk ('16) in their proof of Courcelle's conjecture.

Miraftab, Morin, and Yuditsky ('26) independently proved the same results in the case of path decompositions, with different techniques and significantly better bounds than ours: polynomial instead of double exponential. The path decomposition case turns out to be the only bottleneck in our proof: Combining the two approaches, and the recent polynomial bounds in the Graph Minor Structure Theorem of Gorsky, Seweryn, and Wiederrecht ('25), one obtains that graphs excluding H as a minor have basis number bounded by a polynomial in $|H|$.

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Contributed Talk Abstracts

Thursday, August 27

On Ternary Trees and Fighting Fish

Sen-Peng Eu* Tung-Shan Fu† Yu-Ren Pan‡

Abstract

In a study of random branching surfaces, Duchi–Guerrini–Rinaldi–Schaeffer (Fighting fish, *J. Phys. A: Math. Theor.* 50 (2017) 024002) introduced a combinatorial configuration called *fighting fish* in terms of gluings of unit squares as a generalization of directed convex polyominoes. Fighting fish are counted by the sequence $\frac{2}{(n+1)(2n+1)}\binom{3n}{n}$, $n \geq 1$, which turns out to be the number of some classical objects such as *left ternary trees*, *two-stack-sortable permutations*, *non-separable rooted planar maps* and *synchronized intervals of the Tamari lattices*. Recently, there has been growing interest in bijections among these different families of objects.

In this work, we introduce the notion of *stem cells* of fighting fish and present a new construction of the fighting fish, using a tree structure built on the stem cells. From this perspective, we establish a bijection between ternary trees and fighting fish with a marked strip of cells, which specializes to a direct bijection between left ternary trees and fighting fish. This result contributes a missing link for the interconnection among these objects. Using these results, we obtain a combinatorial enumeration of the fighting fish of size n by establishing an $(n+1)$-to-2 bijection with the ternary trees with n nodes. We present some additional enumerative results including that the fighting fish with a marked tail and the horizontally symmetric fighting fish are equinumerous with the ordered pairs of ternary trees having a total of a prescribed number of nodes.

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Sharp bounds for uniform union-free hypergraphs

Miao Liu* Chong Shangguan† Chenyang Zhang‡

Abstract

For fixed integers $r, t \geq 2$, an r-uniform hypergraph is called t-union-free if any two distinct collections $\mathcal{A}, \mathcal{B} \subseteq \mathcal{F}$ with $1 \leq |\mathcal{A}|, |\mathcal{B}| \leq t$, we have $\cup_{A\in\mathcal{A}} A \neq \cup_{B\in\mathcal{B}} B$. The study of union-free hypergraphs has multiple origins in coding theory, extremal combinatorics, and non-adaptive group testing. In the special case $r = 2$ and $t = 2$, the problem is equivalent to the classical problem of Erdős on determining the maximum number of edges in a $\{C_3, C_4\}$-free graph. Let $U_t(n, r)$ denote the maximum number of edges in an n-vertex t-union-free r-uniform hypergraph.

In this paper, we determine the asymptotic behavior of $U_t(n, r)$, up to a lower-order term, for almost all $t \geq 3$ and $r \geq 3$, with the exception of the cases $\{U_3(n, 2s-1), U_{s+1}(n, s+1), U_{s+1}(n, s+2) : s \geq 2\}$, and $U_4(n, 7)$. In particular, for all $t \geq 2$ and $k \geq 2$, we show that,

$$\lim_{n\to\infty} \frac{U_{t+1}(n, tk)}{n^k} = \frac{1}{k!} \frac{1}{\binom{tk-1}{k-1}}.$$

This significantly advances the understanding of this extremal function. Previously, the asymptotic behavior had been determined only for $U_2(n, 3)$ and $U_2(n, 4)$ by Frankl and Füredi. As a key ingredient of our proof, we establish the existence of near-optimal locally sparse induced hypergraph packings, which is of independent interest.

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Hamiltonicity of $(P_2 \cup kP_1)$-free graphs with many vertices

Masahiro Sanka*

Abstract

Let $k \geq 1$ be an integer. A graph G is $(P_2 \cup kP_1)$-free if G does not contain $P_2 \cup kP_1$ as an induced subgraph, where $P_2 \cup kP_1$ is the graph of order $k+2$ and size 1. In 2019, Ota and the author proved that for $k \leq 3$, every 1-tough and k-connected $(P_2 \cup kP_1)$-free graph is hamiltonian or the Petersen graph, and for $k \geq 4$, every 1-tough and k-connected $(P_2 \cup kP_1)$-free graph with minimum degree at least $\frac{3k-3}{2}$ is hamiltonian. Recently, Hu, Wang and Shen improved the minimum degree condition to at least $\frac{7k-6}{5}$. However, it is not known whether every 1-tough and k-connected $(P_2 \cup kP_1)$-free graph is hamiltonian for $k \geq 4$.

In this talk, we present that the above question has a positive solution for graphs with at least a number of vertices depending on k. More precisely, we show that for $k \geq 4$, every 1-tough and $(k-1)$-connected $(P_2 \cup kP_1)$-free graph on at least $k^2 + k + 1$ vertices is hamiltonian. We will also introduce related results on longest cycles of 1-tough and $(k-1)$-connected $(P_2 \cup kP_1)$-free graphs.

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Between proper and square colorings of sparse graphs

Ilkyoo Choi* Xujun Liu†

Abstract

An *i-independent set* is a set of vertices whose pairwise distance is at least $i+1$. A *proper coloring* (resp. a *square coloring*) of a graph is a partition of its vertices into independent (resp. 2-independent) sets. A *packing* $(1^\ell, 2^k)$*-coloring* of a graph is a partition of its vertices into ℓ independent sets and k 2-independent sets; this is an intermediate coloring between proper coloring and square coloring. We investigate classes of sparse graphs that have a proper $(\ell+1)$-coloring but no packing $(1^\ell, 2^k)$-coloring for any finite k.

The Four Color Theorem states that every planar graph is packing (1^4)-colorable, and Grötzsch's Theorem says every planar graph with girth at least 4 is packing (1^3)-colorable. However, for every fixed k, we construct a planar graph with no packing $(1^3, 2^k)$-coloring and a planar graph with girth 6 that has no packing $(1^2, 2^k)$-coloring. Moreover, for every positive integer ℓ, we completely determine the minimum girth condition $g(\ell)$ for which every planar graph with girth at least $g(\ell)$ has a packing $(1^\ell, 2^{f(\ell)})$-coloring for some finite $f(\ell)$. Our results are actually in terms of maximum average degree.

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On spectrally indistinguishable pseudorandom graphs

Shohei Satake*

Abstract

In recent preprints (arXiv:2511.21351 and 2603.13198), Forey, Fresán, Kowalski, and Wigderson provided explicit constructions of spectrally indistinguishable pseudorandom graphs, whose eigenvalue distributions asymptotically converge to the semicircle distribution. Hence, these graphs are spectrally indistinguishable from random graphs of the corresponding edge density. Moreover, they proved that these explicit graphs do not contain the complete bipartite graph $K_{2,3}$, in sharp contrast to random graphs. This demonstrates that the eigenvalue distribution cannot force the existence of local structures (such as $K_{2,3}$) which appear in random graphs with high probability, thereby exhibiting a “limit” of modern spectral graph theory.

In this talk, we focus on two known families of spectrally indistinguishable pseudorandom graphs, namely, *Li's graphs* and *finite Euclidean graphs*. Specifically, we prove that these graphs do not contain $K_{2,3}$, revisiting them as striking examples of spectrally indistinguishable pseudorandom graphs that evade this local universality. Furthermore, we remark that Li's graphs do not contain the triangle K_3, highlighting that the asymptotic eigenvalue distribution cannot force the existence of K_3 either.

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On pattern-avoiding hypermatrices and higher-dimensional partitions

Sen-Peng Eu* Yi-Lin Lee † Shen-Fu Tsai ‡

Abstract

A $(0,1)$-matrix A *contains* a $(0,1)$-matrix B if A has a submatrix that can be turned into B by flipping any number of 0's to 1's. Otherwise we say that A *avoids* B. In 2021, Brualdi and Cao initiated the study of maximally I_k-avoiding $(0,1)$-matrices (IAMs), that is, $(0,1)$-matrices that avoid I_k (the identity matrix of size k) and flipping any 0 of which makes them contain I_k. They proved that every IAM of size $n_1 \times n_2$ contains the same number of 1's, and these 1's can be decomposed into $k-1$ zigzag paths of lengths $n_1+n_2-1, n_1+n_2-3, n_1+n_2-5, \ldots$. This was later generalized by the third author to d-dimensional $(0,1)$-hypermatrices, where zigzag paths become d-dimensional staircases (or maximal antichains in a strict chain product poset), and I_k consists of k^d entries, each of which is 1 if and only if all of its d coordinates are identical.

In this talk, we present our work as follows.

1. We establish a bijection between 2-dimensional IAMs and plane partitions of a certain size by interpreting the 1's of the IAMs as non-intersecting lattice paths on the square lattice.
2. For every 2-dimensional IAM, the aforementioned zigzag path decomposition is not unique; we show that there are exactly $(k-1)!$ such decompositions.
3. We generalize these results to d-dimensional $(0,1)$-hypermatrices. Specifically, we establish a bijection between $n_1 \times n_2 \times \cdots \times n_d$ IAMs and d-dimensional partitions of a certain size. We show that the 1's can be decomposed into $k-1$ staircases of sizes $\prod_{i\in[d]} n_i - \prod_{i\in[d]}(n_i - 1), \prod_{i\in[d]}(n_i-1)-\prod_{i\in[d]}(n_i-2), \ldots$. Moreover, the number of such decompositions is $(k-1)!$, independent of the dimension d.

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Relative discrepancy of hypergraphs

Diep Luong-Le* Tuan Tran† Dilong Yang‡

Abstract

Given k-uniform hypergraphs G and H on n vertices with densities p and q, their relative discrepancy is defined as $\mathrm{disc}(G, H) = \max \left| |E(G') \cap E(H')| - pq\binom{n}{k} \right|$, where the maximum ranges over all pairs G', H' with $G' \cong G$, $H' \cong H$, and $V(G') = V(H')$. Let $\mathrm{bs}(k)$ denote the smallest integer $m \geq 2$ such that any collection of m k-uniform hypergraphs on n vertices with moderate densities contains a pair G, H for which $\mathrm{disc}(G, H) = \Omega(n^{(k+1)/2})$.

In this talk, we present key steps toward addressing several questions raised by Bollobás and Scott, and establish new results for $\mathrm{bs}(k)$. These include the exact value of $\mathrm{bs}(k)$ for $2 \leq k \leq 13$, as well as a general upper bound $\mathrm{bs}(k) = O(k^{0.525})$, which significantly improves the previous bound $\mathrm{bs}(k) \leq k + 1$ due to Bollobás–Scott.

The case $k = 2$ recovers a result of Bollobás–Scott, which generalizes classical theorems of Erdős–Spencer, and Erdős–Goldberg–Pach–Spencer. The case $k = 3$ also follows from the results of Bollobás–Scott and Kwan–Sudakov–Tran. Our proof combines techniques from linear algebra, Fourier analysis, and extremal hypergraph theory.

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Hamiltonian Properties of 3-Connected Claw-Free Graphs and Line Graphs of 3-Hypergraphs

Kenta Ozeki* Leilei Zhang†

Abstract

Motivated by Thomassen's well-known line graph conjecture, many researchers have explored sufficient conditions for claw-free graphs to be Hamiltonian or Hamilton-connected. In 1994, Ageev proved that every 2-connected claw-free graph with domination number at most 2 is Hamiltonian. In this paper, we extend this line of research to 3-connected graphs by establishing the best possible upper bound on the domination number that guarantees Hamiltonicity. Specifically, we show that, except for some well-defined exceptional graphs, every 3-connected claw-free graph G with domination number at most 5 is Hamiltonian. Furthermore, we prove that, apart from a few exceptional cases, every 3-connected claw-free graph G with domination number at most 4 is Hamilton-connected, thereby generalizing earlier results of Zheng, Broersma, Wang and Zhang and Vrána, Zhan and Zhang. We further investigate the Hamiltonian properties of line graphs of 3-hypergraphs, and prove that every 3-connected line graph of a 3-hypergraph with domination number at most 4 is Hamiltonian.

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New bounds for proper h-conflict-free colourings

Quentin Chuet* Tianjiao Dai† Qiancheng Ouyang‡ François Pirot§

Abstract

A proper k-colouring of a graph G is called h-conflict-free if every vertex v has at least $\min\{h, \deg(v)\}$ colours appearing exactly once in its neighbourhood. Let $\chi_{\mathrm{pcf}}^h(G)$ denote the minimum k such that such a colouring exists. Caro, Petruševski, and Škrekovski conjectured that if G is a connected graph of maximum degree $\Delta \geq 3$, then $\chi_{\mathrm{pcf}}^1(G) \leq \Delta + 1$.

We prove this conjecture holds asymptotically by showing that, for every fixed $h \geq 1$, every graph G of maximum degree Δ satisfies $\chi_{\mathrm{pcf}}^h(G) \leq h\Delta + \mathcal{O}(h \log \Delta)$. This improves a recent result of Liu and Reed in the case $h = 1$. We conjecture that for every $h \geq 1$ and every graph G of maximum degree Δ sufficiently large, the bound $\chi_{\mathrm{pcf}}^h(G) \leq h\Delta + 1$ should hold, which would be tight. When the minimum degree δ of G is sufficiently large, namely $\delta \geq \max\{100h, 2000 \log \Delta\}$, we show that this upper bound can be further reduced to $\chi_{\mathrm{pcf}}^h(G) \leq \Delta + \mathcal{O}\sqrt{h\Delta}$. This improves a recent bound from Kamyczura and Przybyło when $\delta \leq \sqrt{h\Delta}$.

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Clique factors in random samplings of regular graphs

Wanting Sun* Shunan Wei† Donglei Yang‡

Abstract

The robustness of a graph under random vertex sampling provides a natural measure of the persistence of spanning properties. Draganić, Keevash and Müyesser [Int. Math. Res. Not. 2025(14)] resolved a conjecture of Erdős and Faudree [Combin. Probab. Comput. 8 (1999)] by showing that a random vertex subset of an $(n+1)$-regular graph on $2n$ vertices induces a Hamiltonian subgraph with probability at least $1/2$, and conjectured an analogue for the Hajnal–Szemerédi theorem.

We prove this conjecture for all sufficiently large n. More precisely, for every $r \geq 2$, there exists a constant $c > 0$ such that for all sufficiently large n, every $((r-1)n+1)$-regular graph G on rn vertices contains at least $c2^{rn}$ subsets $S \subseteq V(G)$ for which $G[S]$ admits a K_r-factor.

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Sparse Partitions of Graphs with Bounded Clique Number

António Girão* Toby Insley*

Abstract

Given a graph H, say that a graph G is H*-free* if G contains no induced copy of H. A number of conjectures, most notably that of Erdős and Hajnal, claim that H-free graphs exhibit a highly atypical structure. A particularly strong conjecture in this direction was recently raised by Fox, Nguyen, Scott and Seymour. Given a graph G and $\varepsilon > 0$, say that a partition $V(G) = A_1 \cup \cdots \cup A_t$ is ε*-restricted* if either $G[A_i]$ or its complement has maximum degree at most $\varepsilon|A_i|$ for each i. For which graphs H is there some constant $C = C(H) > 0$ such that, for every H-free graph G and any $\varepsilon > 0$, there is an ε-restricted partition of $V(G)$ into at most $(1/\varepsilon)^C$ sets? The mentioned authors showed this to be true when H is the four-vertex path, and asked whether the same can be said when H is a triangle. We answer this question, showing more generally that the above is true when H is any complete graph, and that moreover we may insist the sets making up such a partition are all sparse.

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Prefix-bounded matrices

Nóra A. Borsik* András Frank† Péter Madarasi‡ Tamás Takács§

Abstract

Alternating sign matrices (ASMs) are $(0, \pm 1)$-valued matrices such that the sum of the entries in each row and column equals 1 and in every row and column the non-zero entries alternate in sign. Unifying numerous earlier extensions of ASMs — including partial ASMs, higher-spin ASMs, and sum-majorized matrices — we introduce prefix-bounded matrices (PBMs), which impose lower and upper bounds on the sums of entries in all horizontal and vertical prefixes. We show that the convex hull of PBMs is the intersection of two special generalized polymatroids, proving in greater generality the box total dual integrality of the Behrend–Knight–Striker inequality system, thereby settling a recent conjecture of Edmonds. This structural insight yields a complete feasibility characterization for PBMs under entry and total-sum bounds, encompassing several existence problems concerning ASMs studied by Brualdi and Dahl. Moreover, we prove that the constraint matrix of the natural linear inequality system describing the convex hull of PBMs is a network matrix, which implies that the polyhedron admits a sharpened form of the integer decomposition property, and provides efficient network-flow algorithms for feasibility and optimization problems concerning PBMs. As an application, we confirm an extended form of a conjecture of Brualdi and Dahl, namely that every so-called k-regular ASM decomposes into k pattern-disjoint ASMs.

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Hypergraph independence bounds: from maximum degree to average degree

Jing Yu[*] Junchi Zhang[†]

Abstract

We prove a transfer theorem for hereditary classes of $(r+1)$-uniform hypergraphs. Under a mild regularity hypothesis on a function f, a lower bound of the form $(1-o_\Delta(1))f(\Delta)/\Delta^{1/r}$ for independence number in terms of maximum degree implies the corresponding lower bound of the form $(1-o_d(1))f(d)/d^{1/r}$ in terms of average degree. In the logarithmic case, this recovers and unifies average-degree bounds for linear, locally sparse and uncrowded uniform hypergraphs. We also derive graph-theoretic consequences for locally q-colorable graphs, graphs with bounded local Hall ratio, graphs excluding a fixed cycle, clique, bipartite graph, almost bipartite graph, or 3-colorable graph, and classes defined by local edge-density conditions.

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Induced outerplanar subgraphs in planar graphs

Kengo Enami* Naoki Matsumoto† Takamasa Yashima‡

Abstract

One of the central open problems on planar graphs is the Albertson–Berman conjecture, which states that every planar graph on n vertices contains an induced forest with at least $n/2$ vertices. In this talk, we consider induced outerplanar subgraphs and related notions in plane graphs as a relaxation of induced forests.

Let G be a plane graph (whose planar embedding is fixed). Let $s_o(G)$ denote the maximum number of vertices in an induced outerplanar subgraph of G. Similarly, let $s_{o'}(G)$ (resp. $s_{K_4}(G)$) denote the maximum number of vertices in an induced outerplane (resp. K_4-minor free) subgraph of G. Note that the embedding of G is fixed, an induced outerplanar subgraph may not necessarily appear as an outerplane subgraph in G. We study the relationships among these parameters and obtain several results. As an example, we prove that for every plane graph G of order n, $s_o(G) - s_{o'}(G) \leq \frac{s_o(G)-2}{3} \leq \frac{n-2}{3}$, and we show that this bound is sharp.

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The achromatic number and the pseudoachromatic number of caterpillars

Naoki Matsumoto * Yumiko Ohno†

Abstract

A complete k-coloring (resp., pseudocomplete k-coloring) of a graph G is a proper k-coloring (resp., k-coloring which is not necessarily proper) such that each pair of colors appears on at least one edge. Every $\chi(G)$-coloring of G is a (pseudo)complete $\chi(G)$-coloring; if there is a pair (i, j) of colors which does not appear on any edge, then we can obtain a $(\chi(G) - 1)$-coloring of G by recoloring all vertices with color j by color i, a contradiction. We define the achromatic number (resp., pseudoachromatic number) of G, denoted $\psi(G)$ (resp., $\psi_s(G)$), to be the maximum number k for which G has a complete k-coloring (resp., a pseudocomplete k-coloring).

In general, the difference between $\psi(G)$ and $\psi_s(G)$ for a graph G can be arbitrarily large. On the other hand, Hedetniemi conjectured that for trees T, $\psi_s(T) - \psi(T) \leq 1$. It is known that the problem of determining the achromatic number of trees is NP-hard and we first show that the problem remains NP-hard for trees with pathwidth 2, where pathwidth is the parameter majoring how far between a graph and a path. Then we concentrate on trees with pathwidth 1, called caterpillars. We prove that Hedetniemi's conjecture holds for caterpillars with certain conditions, and we estimate the (pseudo)achromatic number of several specific caterpillars.

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Note on the trace of random walks on pseudorandom graphs

Yaobin Chen*

March, 29, 2026

Abstract

We study the graph-theoretic properties of the trace of random walks on pseudorandom graphs. We show that for any $\varepsilon > 0$, there exists a constant C such that the cover time of an (n, d, λ)-graph G with $d/\lambda \geq C$ is at most $(1+\varepsilon)n \log n$, meaning the expected number of steps needed to reach all vertices at least once is at most $(1+\varepsilon)n \log n$ regardless of the starting vertex. Furthermore, we prove that with high probability, the trace of a random walk of length $(1+\varepsilon)n \log n$ on G is Hamiltonian, regardless of the starting vertex. These results also hold for random d-regular graphs with sufficiently large d. These findings answer two questions proposed by Frieze, Krivelevich, Michaeli, and Peled [PLMS, 2018]. Notably, our results imply a bound on a stronger version of the cover time: with high probability, all vertices are covered after $(1+\varepsilon)n \log n$ steps, regardless of the starting vertex. Our proofs rely on the spectral properties of the adjacency matrix and the graph expansion. All results are asymptotically optimal.

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Room F

Thursday, August 27 | 11:30–12:00

Top Trading Cycles with Random Preferences

Nick Arnosti* Alan Frieze† Pawel Pralat‡

Abstract

We consider exchange economies in which n individuals own distinct objects, and wish to exchange these objects with one another. One example of such a setting is exchange of dorm rooms or subsidized apartments among current tenants: each tenant would be willing to trade their apartment for others that better suit them. Such situations can be represented by a directed graph, with vertices representing individuals, and an edge from i to j if i would be willing to trade its object in return for j's. A set of trades can be represented as a set of vertex-disjoint cycles in this graph.

There are many algorithms that can be used to find a set of trades. If each individual has strict preferences over objects, one well-studied choice is the Top Trading Cycles algorithm (TTC). This algorithm is known to select the unique allocation in the core, and is the only algorithm that is individually rational, Pareto efficient, and strategy-proof.

We study the outcome of TTC on random graphs. We measure the outcome using the *rank distribution*: the fraction of individuals who trade their object for their k^{th} choice, for each value of k. Our random graph model is as follows. We let d_i be the number of objects that i considers *acceptable* (preferable to i's initial object). The values d_i are sampled from a distribution $\mathcal{D}$ on the natural numbers. Given d_i, the preference list of i is sampled uniformly at random among ordered lists of length d_i. Our main contribution is to provide *exact* expressions for the rank distribution, as the market size $n \to \infty$.

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On the density of Kravitz sets

Vsevolod Lev[*] Máté Matolcsi[†] Péter P. Pach[‡] Dániel Varga[§]

Abstract

At the problem session of the Edinburgh meeting on additive combinatorics (2024), Noah Kravitz asked how large a subset $A \subseteq \mathbb{F}_p$ can be such that $A + A - 2A \neq \mathbb{F}_p$, where $A + A - 2A = \{a_1 + a_2 - 2a_3\colon\ a_1, a_2, a_3 \in A\}$.

In the integer setting, Bukh proved that for any finite set $A \subseteq \mathbb{Z}$, and any $\lambda_1, \ldots, \lambda_k \in \mathbb{Z}$ with $\gcd(\lambda_1, \ldots, \lambda_k) = 1$, one has $|\lambda_1 A + \cdots + \lambda_k A| \geq \Big(\sum_{i=1}^{k} |\lambda_i|\Big)|A| - o(|A|)$. Fiz Pontiveros proved that the analogous estimate holds in $\mathbb{F}_p$, provided that the set A is small. For dense sets A, this may not be the case. Fiz Pontiveros also proved that for any nonzero integer λ and $\varepsilon > 0$, there exists $\delta > 0$ such that the following holds: if p is a sufficiently large prime, then there exists a set $A \subseteq \mathbb{F}_p$ with $|A| \geq (1/2 - \varepsilon)p$ such that $|A + \lambda A| \leq (1 - \delta)p$.

In this talk, we prove that if $p > 3$ is a prime and $A \subseteq \mathbb{F}_p$ is a subset of size $|A| > \frac{2}{7}p$, then $A + A - 2A = \mathbb{F}_p$.

Our proof uses a linear programming approach, but we will also discuss a purely combinatorial argument as well.

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[†]HUN-REN Alfréd Rényi Institute of Mathematics, Budapest, Hungary.

[‡]HUN-REN Alfréd Rényi Institute of Mathematics, Budapest, Hungary and Institute for Basic Science (IBS), Daejeon, South Korea.

[§]HUN-REN Alfréd Rényi Institute of Mathematics, Budapest, Hungary.

Room B

Thursday, August 27 | 16:00–16:30

Some exact values of the uniform Turán densities

Hao Lin[*] Guanghui Wang[†] Wenling Zhou[‡] Yiming Zhou[§]

Abstract

The *uniform Turán density* $\pi_{\mathrm{u}}(F)$ of a 3-uniform hypergraph (or 3-graph) F is the supremum of all d such that there exist infinitely many F-free 3-graphs H in which every induced subhypergraph on a linearly sized vertex set has edge density at least d. Determining $\pi_{\mathrm{u}}(F)$ for a given 3-graph F was proposed by Erdős and Sós in the 1980s, yet only a few cases are known. In particular, it remains open whether $1/2$ can occur as a value of π_{u}.

In this paper, we establish a novel connection between Turán-type extremal problems for digraphs and uniform Turán densities of 3-graphs. Using digraph extremal results, we give the first verifiable conditions for 3-graphs F with $\pi_{\mathrm{u}}(F) = (r-1)/r$ and $\pi_{\mathrm{u}}(F) = (r-1)^2/r^2$ for all $r \geq 2$, and identify the corresponding 3-graphs.

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Odd-Cycle-Packing-treewidth and Grid Theorem for odd-minor relation

Mujin Choi *† Maximilian Gorsky *‡ Gunwoo Kim *§ Caleb McFarland ¶
Sebastian Wiederrecht ‖

March 19, 2026

Abstract

We introduce a new graph parameter *Odd-Cycle-Packing-treewidth* (OCP-tw), which decomposes a given graph into pars of bounded odd cycle packing number. We then establish an analogue to the celebrated Grid Theorem of Robertson and Seymour for OCP-tw. Specifically, we identify two infinite families of grid-like graphs whose presence as odd-minors implies large OCP-tw and excluding them as an odd-minor implies bounded OCP-tw. This structural result is constructive and implies a $2^{\mathbf{poly}(k)}\mathbf{poly}(n)$-time parameterized $\mathbf{poly}(k)$-approximation algorithm for OCP-tw. Moreover, we show that the (weighted) Maximum Independent Set problem (MIS) can be solved in polynomial time on graphs of bounded OCP-tw.

*Supported by the Institute for Basic Science (IBS-R029-C1).

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Polynomial χ-boundedness of graphs forbidding specific forests

Yian Xu*

Abstract

A $K^+_{1,1,s}$ is a graph obtained from $K_{1,1,s}$ by adding a new vertex which is exactly adjacent to one vertex of degree $s+1$ in the $K_{1,1,s}$. A $K^+_{2,t}$ is a graph obtained from $K_{2,t}$ by adding a new vertex which is exactly adjacent to one vertex of degree t in the $K_{2,t}$. A family of graphs is said to be polynomial χ-bounded if there exists a polynomial function f such that $\chi(G) \leq f(\omega(G))$ for each graph G in the family. In this talk, we would like to talk about the polynomial χ-boundedness of some classes of graph with forbidden structures related to $K^+_{1,1,s}$ and $K^+_{2,t}$.

*Southeast University, China.

Anti-Ramsey Numbers of Expansions of Graphs

Andrei Kupavskii * Alexander Natalchenko†

Abstract

The anti-Ramsey number $\mathrm{ar}_k(n,\mathcal{H})$, introduced by Erdős, Simonovits and Sós in the 1970s, is the minimum integer c such that in any coloring of the edges of the complete k-uniform hypergraph on n vertices with c (non-empty) colors, we can find a rainbow copy of $\mathcal{H}$ (i.e., its edges have pairwise distinct colors). A hypergraph class that is very well studied in extremal problems are expansions of graphs. Given a 2-graph G, its k-expansion G^+ is a k-uniform hypergraph obtained from G by adding new $k-2$ vertices to each edge in G. The results regarding the anti-Ramsey numbers of expansions include the following. The anti-Ramsey numbers of matchings were studied by Özkahya and Young (2013), Kupavskii and Frankl (2019), Guo, Lu, and Peng (2023). Gu, Li, and Shi considered the anti-Ramsey numbers of paths and cycles (2020). Tang, Li and Yan investigated the anti-Ramsey numbers of 3-stars (2023). In this work, we generalize the previous results and obtain new upper bounds for the anti-Ramsey numbers $\mathrm{ar}_k(n,G^+)$ of expansions. Some of them turn out to be tight.

Theorem 1 *Let $G=(L\cup R,E)$ be a bipartite graph with $|E|\geq 3$ and $|L|=\ell$. Let $1\leq d_1\leq \ldots \leq d_\ell$ be the degrees of vertices in L. We denote $\delta := d_1$ and $\delta' := d_2$. Then, for sufficiently large integers n and k,*

$$\mathrm{ar}_k(n,G^+)\leq \begin{cases} \binom{n}{k}-\binom{n-\ell+2}{k}+\mathrm{ex}_k(n-\ell+2,K^+_{1,\delta'}) \quad +2, & \delta=1,\\ \binom{n}{k}-\binom{n-\ell+1}{k}+\mathrm{ex}_k(n-\ell+1,K^+_{1,\delta-1})+2, & \delta\notin\{1,3\},\\ \binom{n}{k}-\binom{n-\ell+1}{k}+\mathrm{ex}_k(n-\ell+1,K^+_{1,2}) \quad +3, & \delta=3.\end{cases}$$

We use the spread approximation technique introduced by Kupavskii and Zakharov. We will discuss an even more interesting variation of the problem in which, instead of k-sets, we consider other spread domains, such as the family of integer sequences $[n]^k$ or the family of permutations.

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A step toward Chen-Lih-Wu conjecture

Yangyang Cheng* Zhenyu Li † Wanting Sun ‡ Guanghui Wang §

Abstract

An equitable k-coloring of a graph is a proper k-coloring where the sizes of any two different color classes differ by at most one. In 1994, Chen, Lih, and Wu conjectured that for $k \geq 3$, the only connected graphs with maximum degree at most k that admit no equitable k-coloring are the complete bipartite graph $K_{k,k}$ for odd k and the complete graph K_{k+1}. Kierstead and Kostochka proposed a stronger conjecture, relaxing the maximum degree condition to an Ore-type condition. Their conjecture states that for $k \geq 3$, if G is an n-vertex graph such that $d(x) + d(y) \leq 2k$ for every edge $xy \in E(G)$, and G admits no equitable k-coloring, then G contains either K_{k+1} or $K_{m,2k-m}$ for some odd m. We prove that for any constant $c > 0$ and all sufficiently large n, both conjectures hold for every $k \geq cn$. Our proof yields a polynomial-time algorithm that decides whether G has an equitable k-coloring, thereby answering a conjecture of Kierstead, Kostochka, Mydlarz, and Szemerédi when $k \geq cn$.

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On m-partite (di)graphical semiregular representation of finite groups

Jia-Li Du*

Abstract

Extending the well-studied concept of (di)graphical regular representations to bipartite (di)graphs, a Haar (di)graphical representation (HGR) of a group G is a bipartite (di)graph whose automorphism group is isomorphic to G and acts semiregularly with the orbits giving the bipartition. The question of which groups admit an HGR was inspired by a closely related question of Estélyi and Pisanski in 2016, as well as Babai's work in 1980 on poset representations, and has been recently solved by Morris and Spiga. In this talk, I will introduce the m-partite (di)graphical representation with $m \geq 2$, and some related work about semiregular representations of finite groups. This is a joint work with Yan-Quan Feng, Pablo Spiga, Binzhou Xia and Da-Wei Yang.

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On oriented Turán problems

Dániel Gerbner* Xuanrui Hu† Yuefang Sun‡

Abstract

The oriented Turán number of a given oriented graph $\overrightarrow{F}$, denoted by $\mathrm{ex}_{\mathrm{o}}(n, \overrightarrow{F})$, is the largest number of arcs in n-vertex $\overrightarrow{F}$-free oriented graphs. This concept could be seen as an oriented version of the classical Turán number.

In this paper, we first prove several propositions that give exact results for several oriented graphs. In particular, we determine all exact values of $\mathrm{ex}_{\mathrm{o}}(n, \overrightarrow{F})$ for every oriented graph $\overrightarrow{F}$ with at most three arcs and sufficiently large n. After that, we prove a stability result and use it to determine the Turán number of an orientation of C_4. Finally, we prove oriented versions of the random zooming theorem by Fernández, Hyde, Liu, Pikhurko and Wu and the almost regular subgraph theorem by Erdős and Simonovits, and use them to obtain an oriented version of the Füredi-Alon-Krivelevich-Sudakov Theorem, which generalizes the famous KST Theorem.

*HUN-REN Alfréd Rényi Institute of Mathematics, Hungary. Supported by the National Research, Development and Innovation Office - NKFIH under the grant KKP-133819 and by the János Bolyai scholarship.

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Unavoidable pivot-minors in graphs of large rank-depth

Jungho Ahn* Kevin Hendrey† O-joung Kwon‡ Sang-il Oum§

August 17, 2026

Abstract

Shrub-depth and rank-depth are related graph parameters that are dense analogs of tree-depth. We prove that for every positive integer t, every graph of sufficiently large rank-depth contains a pivot-minor isomorphic to a path on t vertices or a graph consisting of two disjoint cliques of size t joined by a half graph. This answers an open problem raised by Kwon, McCarty, Oum, and Wollan in 2021.

*Department of Computer Engineering, Inha University, Incheon, South Korea. Supported by the KIAS Individual Grant (CG095301) at Korea Institute for Advanced Study and Leverhulme Trust Research Project Grant RPG-2024-182 and Inha University Research Grant.

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On the chromatic profile for tripartite graphs and beyond

Yisai Xue*

Abstract

Let H be a graph and let $\delta_\chi(H, r)$ denote the infimum of c such that every H-free graph with minimum degree at least cn is r-colorable. The *chromatic profile* of H is defined to be the values of $\delta_\chi(H, r)$ as r varies. Erdős and Simonovits described this parameter as "too complicated," and it was later highlighted again as a formal open problem by Allen, Böttcher, Griffiths, Kohayakawa, and Morris: determine the chromatic profile of every graph H.

In this paper, we study this problem for the fundamental case of tripartite graphs. We prove that the set of possible values of $\delta_\chi(H, 2)$ for graphs H with $\chi(H) = 3$ is finite and discrete:

$$\{\delta_\chi(H, 2) : \chi(H) = 3\} = \left\{\frac{1}{2}, \frac{2}{5}, \frac{2}{7}, \frac{1}{4}, \frac{2}{9}, \frac{1}{5}, \frac{2}{11}, \frac{1}{6}\right\}.$$

Moreover, we give a complete structural characterization of the graphs H corresponding to each of these threshold values. In addition, we determine $\delta_\chi(H, r)$ for every $r \geq 2$ when H is an edge-critical graph with odd girth $g_{\text{odd}}(H) = \chi(H) = 3$.

Based on the concept of vertex-extendable due to Liu, Mubayi, and Reiher, we define the *vertex-extendable threshold* of H, denoted by $\delta_{\text{ext}}(H, r)$, as the infimum of $c \in (0, 1)$ so that for every H-free graph G on n vertices, the existence of a vertex $v \in V(G)$ with $\chi(G-v) \leq r$, combined with $\delta(G) \geq cn$, implies that G is r-colorable. We showed that

$$\delta_\chi(H, 2) = \max\left\{\delta_\chi(C_{2k+1}, 2), \delta_{\text{ext}}(H, 2)\right\},$$

where H is a color-critical graph with $\chi(H) = 3$ and $g_{\text{odd}}(H) = 2k + 1$ for $k \geq 2$. This is joint work with Bo Ning and Jian Wang.

*Ningbo University, China.

Monochromatic paths and components

Miklós Ruszinkó*

Abstract

The following theorem of Gyárfás [Comm. Comput. Automat. Inst. Hungar. Acad. Sci. 71 (1977)] initiated a lot of research.

Theorem 1 *Every r colored K_n has a monochromatic component with at least $n/(r-1)$ vertices and the equality holds if $(r-1)^2 \mid n$ and there is an affine plane of order $r-1$.*

Füredi [Discrete Math. 75 (1989)] added that an r-colored K_n has a (significantly larger) monochromatic component with at least $n/(r-1-(r-1)^{-1})$ vertices, if an affine plane of order $r-1$ does not exist.

The following problem of Gyárfás [Progr. Math. 285 (2011)] combines large monochromatic component with small diameter. *Given positive numbers n, r. Is there a constant d (perhaps $d=3$) such that in every r-coloring of K_n there is a monochromatic subgraph of diameter at most d with at least $n/(r-1)$ vertices?*

This has been confirmed with $d=5$ by the author [J. Graph Theory 69 (2012)] and with $d=4$ by Letzter [J. Graph Theory 80 (2015)]. The best possible case $d=3$ remains open. We will present some old and new results in this topic with a glance to r-color Ramsey numbers of paths and cycles emphasizing the key role of the Szemerédi's Regularity Lemma as proof technique.

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θ-free graphs: characterization and consequences

Rohinee Joshi* Santhosh Raghul† Ajit Diwan‡ Nishad Kothari§

Abstract

For most problems related to perfect matchings, one may restrict attention to matching covered graphs — connected nonempty graphs in which each edge lies in some perfect matching; Lucchesi and Murty's monograph [Perfect Matchings: A Theory of Matching Covered Graphs, 2024] discusses the extensive literature on them. A cornerstone of this theory is Lovász and Plummer's [Matching Theory, 1986] ear decomposition theorem, which yields interesting open problems in addition to being a fundamental problem-solving tool. We discuss two such open problems below.

By a *bisubdivision* of a graph, we mean a subdivision wherein each edge is replaced by a path of odd length; any bisubdivision of a matching covered graph (except K_2) is also matching covered. A subgraph H of a graph G is *conformal* if $G - V(H)$ has a perfect matching. The aforementioned ear decomposition theorem implies that every matching covered graph (except K_2 and cycles) has a conformal bisubdivision of either θ or of K_4 (or both). This immediately leads to two problems: characterize 'θ-free graphs', and likewise, 'K_4-free' ones. A characterization of planar K_4-free graphs was obtained by Kothari and Murty [*J. Graph Theory*, 2016]; the nonplanar case is open.

We provide a characterization of θ-free graphs that immediately yields a poly-time algorithm for the corresponding decision problem. Our result and its proof relies heavily on a seminal result due to Edmonds, Lovász and Pulleyblank [*Combinatorica*, 1982] pertaining to the tight cut decomposition theory of matching covered graphs. As corollaries, we prove an upper bound of $2n-2$ on the size of θ-free graphs, and we also obtain forbidden minor type characterizations of Pfaffian (θ-free) graphs and 3-edge colorable cubic (θ-free) graphs. Finally, we prove that a cubic graph is θ-free if and only if the length of each conformal cycle is a multiple of four. The Petersen graph and K_4 play key roles in our results.

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Room A

Thursday, August 27 | 17:00–17:30

Shuffle theorem for torus link homology

Donghyun Kim *   Jaeseong Oh †

Abstract

The relationship between Catalan combinatorics, the representation theory of the elliptic Hall algebra, and knot homology has been extensively studied over the past two decades. Building on this work, we prove that the symmetric function $e_{(1^k)}[-MX^{m,n}] \cdot 1$, arising from the elliptic Hall algebra, equals the generating function for k-tuples of cyclic (m,n)-parking functions. This result resolves a conjecture of Gorsky–Mazin–Vazirani and Wilson, establishing that the elliptic Hall algebra governs the Khovanov–Rozansky homology of torus links $T(km,kn)$. Consequently, this provides an affirmative answer to a question posed by Galashin and Lam in the torus link case. As a key step in the proof, we develop a rational analogue of the Shareshian–Wachs involution, which was originally introduced to prove the symmetry property of chromatic quasisymmetric functions. This is based on a cowork with Donghyun Kim.

*Department of Mathematics, Ewha Womans University, Seoul, Korea. Supported by Ewha Womans University Research Grant of 2026.

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Vanishing orders and zero degree Turán densities

Laihao Ding* Hong Liu† Haotian Yang‡

Abstract

For integers $1 \leq \ell < k$, the ℓ-degree Turán density $\pi_\ell(F)$ measures the minimum ℓ-degree threshold that forces a copy of a fixed k-uniform hypergraph F, generalizing both the classical Turán density π_1 and the codegree Turán density π_{k-1}. Motivated by Erdős' characterization of k-graphs with zero Turán density, we study the structural implications of vanishing ℓ-degree Turán density.

We prove for every uniformity $k \geq 3$ that if $\pi_2(F) = 0$, then F admits a 2-vanishing order—a global vertex ordering under which all edges align canonically. This provides a higher-degree analogue of the classical fact that $\pi_1(F) = 0$ forces k-partiteness, and identifies a structural obstruction to vanishing 2-degree Turán density. As an application, we show that, unlike π_1, π_2 accumulates at 0. For $3 \leq \ell \leq k-1$, we also obtain weaker necessary conditions for $\pi_\ell(F) = 0$.

The proof combines random geometric building blocks, a design-theoretic gluing scheme, and random sparsification to reconcile positive 2-degree with local vanishing structure.

*Central China Normal University, Wuhan, China. Supported by the Fundamental Research Funds for the Central Universities (CCNU25JCPT031).

†Institute for Basic Science, Daejeon, South Korea. Supported by IBS-R029-C4.

‡California Institute of Technology, Pasadena, USA. Supported by Seed Fund Program for International Research Cooperation of Shandong University.

A duality theorem for infinite tree-width

Nathan Bowler* Nicola Lorenz* Max Pitz* Jacob Stegemann*

Abstract

We investigate how the known characterisation of graphs of tree-width $\leq \aleph_0$ extends to larger infinite cardinals τ that are regular or singular of cofinality ω. For these cases, we introduce explicit τ-obstructions and show, assuming GCH, that a graph has tree-width $\leq \tau$ exactly when it is τ-unobstructed, that is, when it contains none of these obstructions as minors. This yields a forbidden-minor description of tree-width for these cardinals and extends the $\aleph_0$-case to an infinite setting.

*Universität Hamburg, Hamburg, Germany

Online list version of Hadwiger's conjecture

Yangyan Gu* Xuding Zhu†

Abstract

In 1984, Kostochka and Thomason proved independently that K_t-minor free graphs are $O(t(\log t)^{1/2})$-choosable. This upper bound on the choice number of K_t-minor free graphs was improved by Norin and Postle to $O(t(\log t)^{\beta})$ for any $\beta > 1/4$ in 2023, and further improved by Postle to $O(t(\log\log t)^6)$. If combined with a new density increment theorem of Delcourt and Postle, the argument of Postle shows that the upper bound can be further improved to $O(t(\log\log t)^2)$. We extends theorems concerning list coloring of graphs used in the proof of the above result to online list coloring. As a consequence, we show that every K_t-minor free graph is $O(t(\log\log t)^2)$-paintable.

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On the Erdős-Kleitman problem

Andrey Kupavskii* Georgy Sokolov†

Abstract

Let $e(n, s)$ denote the maximum number of subsets of an n-element set that do not contain s pairwise disjoint sets. The study of $e(n, s)$ was initiated by Erdős in 1960s. The problem of determining $e(n, s)$ is a non-uniform version of the famous Erdős Matching Conjecture. The answer is known in some regimes, but here even a general conjecture is missing. Soon after the problem was stated, Kleitman determined $e(sm - 1, s)$ and $e(sm, s)$ for all integer $m, s \geq 1$. It is apparent from Kleitman's result that the residue of n modulo s plays a crucial role in the problem. Thus, it is convenient to parameterize $n = ms + c, 0 \leq c < s$ and denote $\ell = s - c$. In 2017 Frankl and Kupavskii found $e(ms + s - \ell)$ in the cases $\ell = 2$, $m = 1$ and $\ell < \frac{s+3}{m-3}$.

In this work, we formulate a general conjecture concerning $e(n, s)$ and prove it in a wide range of parameters. It has 4 types of extremal families, one of which was not considered before to the best of our knowledge. Specifically, we prove our conjecture and determine $e(n, s)$ in the following three cases.

- $m \leq 2$ (or, equivalently, $n < 3s$).
- $\ell \leq (\frac{1}{2} - \varepsilon)s$, provided $s \geq s_0(m, \varepsilon)$.
- $s \geq s_0(m, c)$.

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On bipartite holes and k-leaf-connectedness

Ryota Matsubara* Haruhide Matsuda†

Abstract

We adopt the concept of the bipartite hole number, as introduced by McDiarmid and Yolov, and extend the result on Hamilton-connectedness to k-leaf-connectedness by considering this concept in terms of a large minimum degree.

A graph G is *Hamilton-connected* if every two vertices of G are connected by a Hamilton path. For $k \geq 2$, a graph G with vertex set $V(G)$ is said to be *k-leaf-connected* if $|V(G)| > k$ and given any subset S of $V(G)$ with $|S| = k$, G has a spanning tree T such that the set S is the set of endvertices of T. Thus a graph is 2-leaf-connected if and only if it is Hamilton-connected.

An (a,b)-*bipartite-hole* in a graph G consists of two disjoint sets of vertices A and B with $|A| = a$, $|B| = b$, and there is no edge between A and B. The *bipartite hole-number* $\tilde{\alpha}(G)$ is defined as the minimum integer r such that G contains no (a,b)-bipartite hole for positive integers a and b with $a + b = r + 1$. In other words, for any positive integers a and b with $a + b = \tilde{\alpha}(G)$, G contains an (a,b)-bipartite-hole.

McDiarmid and Yolov's theorem states a graph G with at least three vertices is Hamiltonian if the minimum degree $\delta(G) \geq \tilde{\alpha}(G)$. Q. Zhou et al. showed that a graph G on at least three vertices is Hamilton-connected if $\delta(G) \geq \tilde{\alpha}(G) + 1$. As a natural extension, we present the following result: a graph G with at least $k+1$ vertices is k-leaf-connected if $\delta(G) \geq \max\{\tilde{\alpha}(G) + k/2,\ k\}$, where $k \geq 2$ is an integer.

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Contributed Talk Abstracts

Friday, August 28

On the Homotopy Type of Unbalanced Subset Complexes

Mikhail Bludov*

Abstract

Given a collection of points $V = \{v_1, \ldots, v_m\}$ in $\mathbb{R}^d$, a subset $S \subset V$ is called *balanced* if $0 \in \operatorname{conv}(S)$. Balanced collections arise naturally in various areas of combinatorics and can be viewed as generalizations of partitions or perfect fractional matchings. They are also closely related to combinatorial fixed-point theorems.

A typical application of KKMS-type theorems in combinatorics proceeds as follows: one considers a closed (or open) covering of a polytope, assigns points to its faces, and fixes a bijection between these points and the covering sets. Under suitable combinatorial conditions on the covering, one can then conclude the existence of a balanced intersection of sets.

In 2015, Musin introduced a method to associate homotopy invariants to coverings via partitions of unity, deriving the KKM theorem as a consequence of covering obstruction theory. In 2017, Frick and Zerbib established a colorful version of the KKMS theorem, from which they deduced Bárány's Colorful Carathéodory theorem.

For a point set V, consider the simplicial complex of unbalanced subsets

$$\mathcal{K}(V) = \{S \subset V \mid 0 \notin \operatorname{conv}(S)\}.$$

In this work, we describe the homotopy type of $\mathcal{K}(V)$ as well as that of its Alexander dual. In particular, we show that if 0 lies in the interior of $\operatorname{conv}(V)$ and the affine hull of V has dimension d, then $\mathcal{K}(V)$ is homotopy equivalent to the sphere S^{d-1}.

Using this result, we develop an obstruction theory for coverings relative to point configurations and recover classical KKMS-type theorems. As an application, we provide an alternative covering-based proof of the Colorful Carathéodory theorem.

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Almost regular subgraphs under spectral radius constraints

Weilun Xu* Guorong Gao† An Chang‡

Abstract

A graph is called K-almost regular if its maximum degree is at most K times the minimum degree. Erdős and Simonovits showed that for a constant $0 < \varepsilon < 1$ and a sufficiently large integer n, any n-vertex graph with more than $n^{1+\varepsilon}$ edges has a K-almost regular subgraph with $n' \geq n^{\varepsilon\frac{1-\varepsilon}{1+\varepsilon}}$ vertices and at least $\frac{2}{5}n'^{1+\varepsilon}$ edges. An interesting and natural problem is whether there exits the spectral counterpart to Erdős and Simonovits's result. In this talk, we focus on this issue. We verify that for constants $\frac{1}{2} < \varepsilon \leq 1$ and $c > 0$, if the spectral radius of an n-vertex graph G is at least cn^{ε}, then G has a K-almost regular subgraph of order $n' \geq n^{\frac{2\varepsilon^2-\varepsilon}{24}}$ with at least $c'n'^{1+\varepsilon}$ edges, where c' and K are constants depending on c and ε. Moreover, for $0 < \varepsilon \leq \frac{1}{2}$, there exist n-vertex graphs with spectral radius at least cn^{ε} that do not contain such an almost regular subgraph. Our results have a wide range of applications in spectral Turán-type problems.

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New conditions ensuring specified factors of graphs

Masaki Kashima*

2026/03/19

Abstract

For positive integers a and b with $a \leq b$, an $[a, b]$-factor of a graph G is a spanning subgraph F of G with $a \leq \delta(F) \leq \Delta(F) \leq b$. In particular, when $a = b = k$ for some positive integer k, a $[k, k]$-factor of G is called a k-factor of G. A pseudo $[a, b]$-factor of G is a spanning subgraph F of G such that every component of F is either isomorphic to K_1 or K_2, or has the minimum degree at least a and the maximum degree at most b. A pseudo k-factor is defined similarly.

As generalizations of results on $[2, b]$-factors of graphs, Bekkai and Kouider (2009) showed that every graph G has a pseudo 2-factor with at most $\alpha(G) - \delta(G) + 1$ components that are isomorphic to K_1 or K_2, and Bekkai (2014) showed that for any $b \geq 4$, every graph G has a pseudo $[2, b]$-factor with at most $\alpha(G) - \left\lfloor \frac{b}{2}(\delta(G) - 1) \right\rfloor$ components that are isomorphic to K_1 or K_2.

We further generalize these results by considering the following function $f_b(G)$ defined from a value on each independent set. For a graph G and a set S of vertices, let $\delta(S)$ denote the minimum degree of vertices in S. For a positive integer $b \geq 2$ and a graph G, let

$$f_b(G) := \max \left\{ |I| - \left\lfloor \frac{b}{2}(\delta(I) - 1) \right\rfloor \,\middle|\, I \text{ is an independent set of } G \right\}.$$

We show that for any $b \geq 2$, every graph G has a pseudo $[2, b]$-factor with at most $f_b(G)$ components that are K_1 or K_2, and this bound on the number of K_1, K_2 components is tight. As $f_b(G) \leq \alpha(G) - \left\lfloor \frac{b}{2}(\delta(G) - 1) \right\rfloor$ and the gap can be arbitrarily large, this result strictly improves the bounds in both Bekkai and Kouider (2009) and Bekkai (2014). Note that our result covers the case $b = 3$ as well, which is missing in previous results. Furthermore, as a corollary of the main result, we obtain a sufficient condition for a graph to have a $[2, b]$-factor, which relaxes a known sufficient condition by Kouider and Lonc (2004).

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Hadwiger's Conjecture for {co-claw, co-gem}-free graphs and {fork, antifork}-free graphs

Daniel Carter* Jung Hon Yip†

Abstract

We prove Hadwiger's Conjecture for {co-claw, co-gem}-free graphs and {fork, antifork}-free graphs, where the *co-claw* is the disjoint union of a triangle and a vertex, the *co-gem* is the disjoint union of a 4-vertex path and a vertex, the *fork* is obtained from $K_{1,3}$ by subdividing one of the edges, and the *antifork* is the complement of the fork. The {co-claw, co-gem}-free graphs include the complements of line graphs of triangle-free multigraphs. In fact, we prove a stronger result: every {co-claw, co-gem}-free graph G has a $K_{\chi(G)}$-model where each branch set has size at most 2, and every {fork, antifork}-free graph G has a $K_{\chi(G)}$-model where at most one branch set has size greater than 2.

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Boolean lattice without small rainbow subposets

Gyula O.H. Katona* Yaping Mao† Kenta Ozeki‡ Zhao Wang§ Gang Yang¶

Abstract

A Boolean lattice $\mathcal{B}_n = (2^X, \leq)$ is the power set of an n-element ground set X equipped with inclusion relation. For two posets $\mathcal{P}$ and $\mathcal{Q}$, we say that $\mathcal{Q}$ contains an *induced copy* of $\mathcal{P}$ if there exists an injection $f : \mathcal{P} \to \mathcal{Q}$ such that $f(X) \leq f(Y)$ if and only if $X \leq Y$ in $\mathcal{P}$. An exact k-coloring is exact if all colors are used at least once. For posets $\mathcal{Q}$ and $\mathcal{P}$, the *Boolean Gallai-Ramsey number* $\mathrm{GR}_k(\mathcal{Q} : \mathcal{P})$ is defined as the smallest n such that any exact k-coloring of the sets in $\mathcal{B}_n$ contains either a rainbow induced copy of $\mathcal{Q}$ or a monochromatic induced copy of $\mathcal{P}$ and the *Boolean rainbow Ramsey number* $\mathrm{RR}(\mathcal{Q} : \mathcal{P})$ is defined as the smallest n such that any coloring of the sets in $\mathcal{B}_n$ contains either a rainbow induced copy of $\mathcal{Q}$ or a monochromatic induced copy of $\mathcal{P}$.

In this paper, we first study the structural properties of Boolean lattice without rainbow induced copy of small posets. As the application of these results, we give exact values and some bounds of Boolean Gallai-Ramsey numbers and Boolean rainbow Ramsey numbers, which improve a result of Chen, Cheng, Li, and Liu in 2020 and give an answer of a question proposed by Chang, Gerbner, Li, Methuku, Nagy, Patkós, and Vizer in 2022.

Keywords: Boolean Gallai-Ramsey number; Boolean rainbow Ramsey number; Boolean lattice

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Dowling's polynomial conjecture for independent sets of matroids

Shiqi Cao* Keyi Chen† Yitian Li‡ Yuxin Wu§

Abstract

The celebrated Mason's conjecture states that the sequence of independent set numbers of any matroid is log-concave, and even ultra log-concave. The strong form of Mason's conjecture was independently solved by Anari, Liu, Oveis Gharan and Vinzant, and by Brändén and Huh. The weak form of Mason's conjecture was also generalized to a polynomial version by Dowling in 1980 by considering certain polynomial analogue of independent set numbers.

Given a matroid $M = (E, \mathcal{I})$ with $|E| = n$, for any $0 \leq k \leq r_M$ define

$$f_k(M) = \sum_{I \in \mathcal{I},\, |I|=k} \left(\prod_{x_i \in I} x_i \right).$$

Dowling conjectured that:

$$f_k^2(M) \geq f_{k-1}(M)\, f_{k+1}(M)$$

holds for any $0 < k < r_M$, where the partial order between two polynomials $f, g \in \mathbb{R}[x_1, x_2, \ldots, x_n]$ is defined by $f \geq g$ if $f - g$ is a polynomial with nonnegative coefficients. It is clear that Dowling's polynomial conjecture implies the weak form of Mason's conjecture. In this paper we completely solve Dowling's polynomial conjecture by using the theory of Lorentzian polynomials. Moreover, we prove a stronger result. For any matroid M of rank r_M,

$$f_l^p(M) \geq \left(1 + \frac{1}{(p-1)l}\right)\left(1 + \frac{2}{(p-1)l}\right) \cdots \left(1 + \frac{p-1}{(p-1)l}\right) f_{l+1}^{p-1}(M) f_{l-p+1}(M)$$

holds for any integers $p \geq 2$ and $p - 1 < l < r_M$.

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Multi-extremal Betti numbers

Daniel McGinnis*

Abstract

Let $S = k[X_1 \cup \cdots \cup X_d]$ where the X_i are disjoint sets of variables and k is a field. Then S has a natural $\mathbb{Z}^d$ grading. If $M = \bigoplus M_{(a_1,\ldots,a_d)}$ is a $\mathbb{Z}^d$-graded finitely generated S-module then $M_i = \bigoplus M_{(0,\ldots,a_i,\ldots,0)}$ is a $\mathbb{Z}$-graded S_i-module where $S_i = k[X_i]$. The multi-extremal Betti numbers of M are "built" from the extremal Betti numbers of the M_i; we will discuss the precise definition in the talk. Roughly speaking, the multi-extremal Betti numbers correspond to elements in the Koszul homology of M that can be written as the tensor product of elements in the Koszul homology of the M_i corresponding to their respective extremal Betti numbers.

We show that multi-extremal Betti numbers are preserved under taking the multi-graded generic initial ideal. As an application, we describe a relationship between the topological Betti numbers of a colored simplicial complex Δ and the topological Betti numbers of the simplicial complex obtained by applying colored algebraic shifting to Δ.

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Room B

Friday, August 28 | 11:00–11:30

On the Number of Triangles in K_4-Free Graphs

Jialin He* Jie Ma† Yan Wang‡ Chunlei Zu§

Abstract

Erdős asked whether for any n-vertex graph G, the parameter $p^*(G) = \min \sum_{i\geq 1}(|V(G_i)|-1)$ is at most $\lfloor n^2/4 \rfloor$, where the minimum is taken over all edge decompositions of G into edge-disjoint cliques G_i. In a restricted case, Győri and Keszegh [Combinatorica, 37 (6) (2017), 1113–1124] proved that $p^*(G) \leq \lfloor n^2/4 \rfloor$ for all K_4-free graphs G. Motivated by their proof approach, they conjectured that for any n-vertex K_4-free graph G with e edges, and any greedy partition P of G of size r, the number of triangles in G is at least $r(e-r(n-r))$. If true, this would imply a stronger bound on $p^*(G)$. We disprove their conjecture by constructing infinitely many counterexamples with arbitrarily large deviations. We further establish a corrected lower bound on the number of triangles in such graphs, which recovers the conjectured bound once the small counterexamples we identify are excluded.

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Partitions of a bipartite graph into cycles containing specified paths

Shuya Chiba * Hajime Matsumura † Koshin Yoshida ‡

Abstract

The classical Ore's theorem [Amer. Math. Monthly, 1960] states that every graph G of order $n \geq 3$ with $\sigma_2(G) \geq n$ is hamiltonian, where $\sigma_2(G)$ denotes the minimum degree sum of two non-adjacent vertices in G. Brandt, Chen, Faudree, Gould and Lesniak [J. Graph Theory, 1997] generalized this result by considering a partition of a graph into a specified number of cycles as follows: For an integer $k \geq 1$, if G is a graph of order $n \geq 4k$ and $\sigma_2(G) \geq n$, then G has k vertex-disjoint cycles $C_1, \ldots, C_k$ such that $\bigcup_{1 \leq i \leq k} V(C_i) = V(G)$. Related to this result, Matsubara and Matsumura [Discuss. Math. Graph Theory, 2008] gave a sharp $\sigma_2(G)$ condition for partitioning a graph G into a specified number of cycles each of which contains exactly one component of a pre-specified union of vertex-disjoint paths (i.e., a pre-specified linear forest).

Along this line, in this study, we consider degree sum conditions for partitioning a bipartite graph into a specified number of cycles. In particular, we give the following result. For a bipartite graph G, let $\sigma_{1,1}(G)$ be the minimum degree sum of two non-adjacent vertices in different partite sets of G. Let $k \geq 1$ be an integer, $p_1, \ldots, p_k$ be positive integers and G be a balanced bipartite graph of order $2n$, where $n \geq \sum_{1 \leq i \leq k} (\lceil p_i/2 \rceil + 2)$. Assume that if $k = 1$ and p_1 is even, then $\sigma_{1,1}(G) \geq n + p_1/2 + 1$; if $k \geq 3$, $p_i \geq 3$ for $1 \leq i \leq k$, and p_j is odd for some j, then $\sigma_{1,1}(G) \geq n + \sum_{1 \leq i \leq k} \lceil p_i/2 \rceil - 1$; otherwise, $\sigma_{1,1}(G) \geq n + \sum_{1 \leq i \leq k} \lceil p_i/2 \rceil$. We show that for any vertex-disjoint paths $P_1, \ldots, P_k$ of order $p_1, \ldots, p_k$ in G, G has k vertex-disjoint cycles $C_1, \ldots, C_k$ such that $\bigcup_{1 \leq i \leq k} V(C_i) = V(G)$ and C_i passes through P_i for $1 \leq i \leq k$.

In this talk, we will show the sharpness of the $\sigma_{1,1}(G)$ condition in the above result and also outline the main idea in the proof.

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All graphs are majority 3-choosable

Jan Ouborny* Max Pitz†

Abstract

A majority colouring of a graph G is a vertex colouring in which every vertex has at least as many neighbours of a different colour as neighbours of its own colour. Majority choosability generalises this by requiring such a colouring to exist from any assignment of colour lists of a given size. The Unfriendly Partition Conjecture asserts that every countable graph is majority 2-colourable; for uncountable graphs, Shelah and Milner showed that majority 3-colourability always holds but 2-colourability may fail.

Anholcer, Bosek, and Grytczuk proved that every countable graph is majority 4-choosable, and Haslegrave improved this to majority 3-choosable for countable graphs, conjecturing that the same holds for all graphs. We confirm this conjecture: every graph is majority 3-choosable.

For the countable case, we follow Haslegrave's strategy. A preprocessing lemma reduces the lists from size 3 to size 2 while ensuring that for every colour c and every relevant infinite neighbourhood, infinitely many vertices exclude c from their sublist. A greedy colouring of finite approximations, chosen to maximise cross-edges, is then extended to all of B by a compactness argument.

For the uncountable case, we adapt the transfinite approach of Shelah and Milner. We enlarge the vertex set to a well-structured saturated set of the same cardinality, then analyse the boundary of its closure. A lemma of Shelah-Milner, combined with the induction hypothesis applied at smaller cardinalities, allows us to colour the boundary vertices so that all critical vertices become happy. The countable and uncountable cases are then unified by transfinite induction on the size of the uncoloured part, completing the proof.

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A lower bound for the coefficients of denormalized Lorentzian Laurent series and applications

Maryam Mohammadi Yekta*

Abstract

The theory of log-concave polynomials has been developed to study objects and problems in Combinatorics and other subfields of mathematics. One particular class of log-concave polynomials called *Lorentzian polynomials* have been used to prove conjectures on the log-concavity of various combinatorial sequences. For instance, they played a central role in the proof of the strongest form of Mason's log-concavity conjecture on the independent sets of matroids.

Another major thread of applications of log-concave polynomials lies in bounding combinatorial quantities via *Gurvits' capacity method*. This method involves bounding coefficients of log-concave polynomials via an optimization problem. This method has been applied to bound the permanent, mixed discriminant, mixed volume, integer flows of graphs, contingency tables, bipartite graph matchings, distribution probabilities related to the traveling salesman problem, and beyond.

Denormalized Lorentzian (DL) polynomials is a class of log-concave polynomials which have received recent attention. They include important combinatorial polynomials, including Schur polynomials, Schubert polynomials, and conjecturally more. We develop *denormalized Lorentzian (DL) Laurent series*. This class is the natural generalization of DL polynomials to homogeneous Laurent series, with the benefit of capturing a number of combinatorial generating series, including the Kostant partition function for integer flows of directed graphs. We prove new coefficient bounds for these Laurent series using Gurvits' capacity method. Applications of our new results include new bounds for integral flows on general directed acyclic graphs and new bounds for the dimensions of weight spaces of $\mathfrak{sl}_{n+1}(\mathbb{C})$ Verma modules. Additionally we are able to generalize and improve similar bounds on contingency tables.

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Packing 4-cliques in edge-weighted graphs

Donglei Yang*

Abstract

A celebrated result of Hajnal and Szemerédi in 1970 gives the best possible bound on the minimum degree of a graph which guarantees the existence of K_r-factors (the $r = 3$ case was previously obtained by Corrádi and Hajnal. Balogh, Kemkes, Lee and Young proposed a variant of Hajnal-Szemerédi Theorem in edge-weighted graphs, asking for the minimum (weighted) degree condition for heavy K_r-factors. The basic triangle case was resolved by Balogh, Molla and Sharifzadeh in 2016. We resolve the next open case (i.e. 4-clique), and our method may shed light on a full resolution of a conjecture of Balogh–Kemkes–Lee–Young.

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On graphs without cycles of length 0 modulo 3 or 4 modulo 6

Takahiro Ueoro *

Abstract

Let $\mathcal{F}$ be a family of graphs. A graph is said to be $\mathcal{F}$-free if it contains no subgraph isomorphic to any member of $\mathcal{F}$. For a positive integer n, we denote by $ex(n, \mathcal{F})$ the maximum number of edges in an n-vertex $\mathcal{F}$-free graph. For integers $k > \ell \geq 0$, a cycle whose length is congruent to ℓ modulo k is called an (ℓ mod k)-cycle. Let $\mathcal{C}_{\ell \bmod k}$ denote the family of all (ℓ mod k)-cycles.

When $(k, \ell) \equiv (0, 1) \pmod 2$, it is known that $ex(n, \mathcal{C}_{\ell \bmod k}) = n^2/4$ for sufficiently large n. When $(k, \ell) \not\equiv (0, 1) \pmod 2$, it is known that $ex(n, \mathcal{C}_{\ell \bmod k}) = O(n)$. However, the exact value of $ex(n, \mathcal{C}_{\ell \bmod k})$ is known only for a limited number of pairs (k, ℓ).

In this talk, we consider the problem of forbidding two families of cycles simultaneously. We prove that $ex(n, \mathcal{C}_{0 \bmod 3} \cup \mathcal{C}_{4 \bmod 6}) \leq (11/8)n - 7/4$, and we also characterize the extremal graphs attaining equality.

The proof proceeds by showing that every $(\mathcal{C}_{0 \bmod 3} \cup \mathcal{C}_{4 \bmod 6})$-free graph is planar. We then estimate the numbers of 5-faces and 7-faces and apply Euler's formula.

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Homomorphism bounds for $(K_4, -)$-minor-free signed graphs of negative girth k

Luis Kuffner* Reza Naserasr†

Abstract

The relation between minor theory and coloring has been at the heart of graph theory, with the caveat that forbidding a given graph as a minor can only result in a sparse family of graphs. The first to address this limitation was Catlin, who proved in 1979 that if a graph has no odd-K_4, then it is 3-colorable. This is much stronger, for the class of odd-K_4-free graphs is not sparse.

Putting this in the context of signed graphs, Gerards and Seymour independently conjectured that if a signed graph $(G, -)$ (where all edges are negative) has no $(K_t, -)$-minor, then $\chi(G) \leq t-1$. The conjecture has recently been disproved by Kühn, Sauermann, Steiner, and Wigderson, but it remains a key question to determine the best function $f(t)$ that replaces the bound $t-1$.

To better address this and similar questions, notions of coloring and homomorphisms of signed graphs are developed. In this work then we present a necessary and sufficient condition for a signed graph $\widehat{B}$, whose negative girth (the length of its shortest negative cycles) is at least k, to be a homomorphic image of every signed graph of negative girth at least k that has no $(K_4, -)$-minor.

As an application, we show that if a signed graph (G, σ) has no $(K_4, -)$-minor, then its fractional balanced chromatic number is $\frac{l}{l-1}$ where l is the negative girth of (G, σ).

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On the problem of Füredi concerning (k, L)-systems

Andrey Kupavskii[*] Nikolai Terekhov[†]

Abstract

For an integer k and a set of integers $L \subseteq \{0, \dots, k-1\}$, a (k, L)-*system* is a family of k-element sets such that the intersection size of any two distinct sets from the family belongs to L. Deza, Erdős, and Frankl [Proc. Lond. Math. Soc. (3) 36 (1978)] introduced the quantity $m(n, k, L)$, which denotes the maximum size of a (k, L)-system on a ground set of n elements. For each pair (k, L), there is an associated integer parameter, called the *rank*, such that $m(n, k, L) = O(n^{\mathrm{rk}(k,L)})$. A conjecture of Füredi on (k, L)-systems [London Math. Soc. Lecture Note Ser. 166 (1991)] states that there also exists a lower bound of the form $m(n, k, L) = \Omega(n^{\mathrm{rk}(k,L)-1+\varepsilon_{k,L}})$.

In this work, we study a more fine-grained version of the class of (k, L)-systems: $\mathcal{M}$-systems, where $\mathcal{M}$ is a closure system (or an intersection-closed system). We show that the analogue of the conjecture of Füredi is in general false for $\mathcal{M}$-systems. To do so, we develop a new entropy-based upper bound that is of independent interest. It improves upon the upper bound in terms of $\mathrm{rk}(k, L)$ and, in some cases, yields a non-integer upper bound on the asymptotic exponent of $m(n, k, L)$.

A particularly important closure system is the family of flats (or subspaces) of a matroid. A natural conjecture asserts that, in this case, $m(n, \mathrm{Flats}(M)) = \Theta(n^{\mathrm{rk}(M)})$, where $m(n, \mathcal{M})$ denotes the maximum size of a $\mathcal{M}$-system on a ground set of n elements. Our entropy-based upper bound shows that this conjecture fails for the Vámos matroid.

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On matching preclusion sets in weighted graphs

Yoshio Sano*

Abstract

The notion of matching preclusion was introduced by Brigham *et al.* [Congr. Numer. 174 (2005)]. Let G be a graph with an even number of vertices. A *perfect matching* in G is a set M of edges such that every vertex is incident to exactly one edge in M. A set F of edges of G is called a *matching preclusion set* if $G - F$ has no perfect matching. The *matching preclusion number* $\mathrm{mp}(G)$ of a graph G is defined to be the minimum cardinality of a matching preclusion set in G. Lin and Zhang [Int. J. Comput. Math.: Comput. Syst. Theory 1 (2016)] gave a 0-1 linear integer programming formulation (MP) for finding the matching preclusion numbers of graphs. Lin and Zhang [J. Comb. Optim. 39 (2020)] defined the *fractional matching preclusion number* $\mathrm{mp}_f(G)$ of a graph G as the optimal objective value of a linear programming relaxation of the problem (MP).

In this talk, we consider the *weighted matching preclusion number* $\mathrm{wmp}(G_w)$ for an edge-weighted graph G_w as the minimum of the total weights of edges in a matching preclusion set in G_w, and also consider the *fractional weighted matching preclusion number* $\mathrm{wmp}_f(G_w)$ of an edge-weighted graph G_w.

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Exploring the science fiction

Donghyun Kim*		Jaeseong Oh†

Abstract

We introduce the Macdonald piece polynomial $\mathrm{I}_{\mu,\lambda,k}[X;q,t]$, a broad generalization of the Macdonald intersection polynomial from the science fiction conjecture of Bergeron and Garsia.

We first present an extension of the science fiction conjecture using $\mathrm{I}_{\mu,\lambda,k}[X;q,t]$. We then derive ∇s_λ algebraically from $\mathrm{I}_{\mu,\lambda,k}[X;q,t]$, and discuss how we can naturally obtain the reformulation of the Loehr–Warrington formula from $\mathrm{I}_{\mu,\lambda,k}[X;q,t]$ combinatorially. In particular, this provides an elementary proof of the Loehr–Warrington conjecture, previously proven by Blasiak–Haiman–Morse–Pun–Seelinger.

Additionally, we demonstrate the utility of this reformulation of the Loehr–Warrington formula, by connecting to Wilson's formula regarding $\nabla e_{(1^n)}$.

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Turán problems for simplicial complexes

Maria Axenovich* Dániel Gerbner† Dingyuan Liu* Balázs Patkós†

Abstract

An abstract simplicial complex $\mathbf{F}$ is a non-uniform hypergraph without isolated vertices whose edge set is closed under taking subsets. The extremal number $\mathrm{ex}(n, \mathbf{F})$ is defined as the maximum number of edges in an $\mathbf{F}$-free simplicial complex on n vertices. Although Turán-type problems for simplicial complexes have long appeared in extremal set theory, a systematic study of $\mathrm{ex}(n, \mathbf{F})$ was initiated only recently by Conlon, Piga, and Schülke. In contrast to uniform hypergraphs, even the order of magnitude of $\mathrm{ex}(n, \mathbf{F})$ remains unknown for most simplicial complexes.

In this talk, we present a general framework for estimating $\mathrm{ex}(n, \mathbf{F})$ via generalised Turán numbers of associated hypergraphs. Using this approach, we determine the asymptotic behaviour, and in some cases the exact value, of $\mathrm{ex}(n, \mathbf{F})$ for large classes of simplicial complexes, extending a result of Conlon, Piga, and Schülke. Several open problems concerning the potential behaviour of $\mathrm{ex}(n, \mathbf{F})$ are formulated.

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Matching Minors: a sequel to the results of Lovász and Plummer

Ravindra Pawar* Nishad Kothari†

Abstract

A connected graph is *matching covered* if each edge lies in some perfect matching. There is extensive literature on this class; one may refer to the recent monograph 'Perfect Matchings: A Theory of Matching Covered Graphs' by Lucchesi and Murty (2024). One of the fundamental pillars of this subject is the tight cut decomposition theory. In particular, Lovász (JCT-B, 1987) established that every matching covered graph G admits a unique decomposition into special ones called *bricks* (nonbipartite) and *braces* (bipartite); we use $b(G)$ to denote its number of bricks. We remark that G is bipartite if and only if $b(G) = 0$.

Certain containment notions such as minors and topological minors play a key role in graph theory. Likewise, there are containment notions that preserve the property of being matching covered. One of them, 'conformal minors', is intrinsically related to the ear decomposition theory of matching covered graphs; Lovász and Plummer showed that every such graph, except K_2 and cycles, contains either θ or K_4 as a conformal minor, where θ is the 3-regular graph on two vertices. Moreover, Lovász proved that every nonbipartite matching covered graph (or equivalently, those with $b \geq 1$) contains either K_4 or triangular prism $\overline{C_6}$ as a conformal minor. It is worth noting that, in the case of nonbipartite matching covered graphs, Lovász's result implies the aforementioned result of Lovász and Plummer; in this sense, the former may be viewed as a refinement of the latter that is applicable to a smaller class of graphs.

In the same spirit, we present a further refinement of Lovász's result that is applicable to those matching covered graphs that satisfy $b \geq 2$. For technical reasons, it is imperative to use a weaker notion of containment, 'matching minors'. In this article, we prove that every such graph contains one of 16 graphs as a matching minor; each of these 16 graphs has the (desired) properties that $b = 2$ whereas every proper matching minor satisfies $b \leq 1$. As a corollary of our main result, we establish Dalwadi's conjecture (2024), which states that every matching covered graph with $b \geq 2$ contains a bisubdivision of $K_{2,3}$ as a subgraph.

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Induced-packing variants of the Erdős-Pósa theorem

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Abstract

An *induced packing* of cycles in a graph is a set of vertex-disjoint cycles with no edges between them. We generalise the classic Erdős-Pósa theorem to induced packings of cycles. More specifically, we show that there exist a function $f(k) = \mathcal{O}(k \log k)$ such that every graph contains either an induced packing of k cycles, or a set X of vertices with $|X| \leq f(k)$ whose closed neighbourhood hits all cycles. We further extend this result to long cycle subgraphs.

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Improved bounds for loose odd cycle densities

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Abstract

While Sidorenko's conjecture remains open, its hypergraph generalisation is known to be false since Sidorenko's own work in the 1990s. Specifically, the homomorphism density $t(C^{(r)}_{2\ell+1}, G)$ of an r-uniform loose cycle in a hypergraph G of density p can be strictly less than $p^{2\ell+1}$. Recently, Nie and Spiro have focused on estimating the optimal exponent $s = s(r,\ell)$ such that $t(C^{(r)}_{2\ell+1}, G) \geq p^{2\ell+1+s}$, extending an earlier work of Fox, Sah, Sawhney, Stoner, and Zhao that settled the case $(r,\ell) = (3,1)$.

We significantly improve the recent upper bound of $s(r,\ell) \leq \frac{4\ell-1}{(r-1)(2\ell+1)-3}$ due to Deng, Han, Nie, and Spiro by proving that $s(r,\ell) \leq \frac{1}{r-2}$. Furthermore, for the r-expansion $\mathrm{E}^r(H)$ of any k-graph H, we prove $t(\mathrm{E}^r(H), G) \geq p^{e(H)+C_H/r}$ for a constant C_H, providing an approximate positive answer to a conjecture of Nie and Spiro.

We also address an interesting case of a general question raised by Blekherman, Raymond, Razborov, and Wei, asking whether the tightness of an inequality of the form $t(H,G) \geq p^s$ is evidenced by a (hyper)graph G with fixed density $p \in (0,1)$. We answer this in the negative for $H = C^{(3)}_3$ by obtaining nontrivial lower-order terms for $t(C^{(3)}_3, G)$ beyond p^4 for any fixed $p \in (0,1)$.

Our proofs utilise information-theoretic tools, including Pinsker's inequality and a novel hypergraph generalisation of Szegedy's edge-vertex transitivity reduction on Sidorenko's conjecture, which may be of independent interest.

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On Reconfiguration Graphs Induced by Rainbow Spanning Trees

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Abstract

We consider edfe-colored multigraph. An edge-colored graph is called *rainbow* if all of its edges have distinct colors. For two rainbow spanning trees T and T' of an edge-colored graph G, we say that T' is obtained from T by an *edge flip* if T' is obtained from T by deleting an edge $e' \in E(T)$ and adding an edge $f \notin E(T)$. The following decision problem arises naturally in combinatorial reconfiguration.

<u>RAINBOW SPANNING TREE RECONFIGURATION</u>
Input: An edge-colored graph G and two rainbow spanning trees T and T' of G.
Output: YES if there is a sequence of rainbow spanning trees $\langle T = T_0, T_1, \ldots, T_m = T' \rangle$ such that T_i is obtained from T_{i-1} by an edge flip for $1 \le i \le m$ and NO otherwise.

It is not known whether this problem can be solved in polynomial time.

To study this problem, for an edge-colored graph G, we consider the *reconfiguration graph* $\mathcal{G}(G)$ whose vertices correspond to rainbow spanning trees of G, and rainbow spanning trees are adjacent in $\mathcal{G}(G)$ if and only if one is obtained from the other by a single edge flip.

We restrict our attention to the following class of graphs. Let $\mathcal{H}_n$ be the set of edge-colored graphs G with n vertices such that for each color c in G, the subgraph induced by the edges of color c is a connected spanning subgraph.

For $G \in \mathcal{H}_n$, we investigate structural properties of the reconfiguration graphs $\mathcal{G}(G)$, including its connectivity, diameter, and circumference. We also discuss theoretical motivations for studying this reconfiguration problem.

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Counting lattice points in generalized permutohedra from A to B

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Abstract

We derive a formula for the number of lattice points in type B generalized permutohedra, providing a concise alternative to the formula obtained recently by Eur, Fink, Larson, and Spink as a result from a study of delta-matroids. Our approach builds upon the existing framework and techniques introduced by Postnikov in his work on type A generalized permutohedra, a family of polytopes interconnected with many mathematical concepts such as matroids and Weyl groups. In particular, we express the number of lattice points in type B generalized permutohedra in terms of Postnikov's notion of G-draconian sequences, from which their Ehrhart polynomials and volume formula follow as consequences.

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A Positive Instance of Scott's Conjecture on Induced Subdivisions

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Abstract

For a graph G, $\chi(G)$ denotes the chromatic number of G and $\omega(G)$ denotes the size of the largest clique in G. A hereditary class of graphs is called χ-bounded if there is a function f such that for each graph G in the class, $\chi(G) \leq f(\omega(G))$.

Scott (1997) conjectured that for every graph H, the class of graphs which do not contain any subdivision of H as an induced subgraph is χ-bounded. He proved his conjecture when H is a tree and when H is the complete graph on four vertices, K_4. Esperet and Trotignon (2019) proved that the conjecture holds when H is K_4 with one edge subdivided once.

Scott's conjecture was disproved by Pawlik et al. (2014). Chalopin et al. (2016) gave more counterexamples including the graph obtained from K_4 by subdividing each edge of a 4-cycle once.

We prove that the conjecture holds when H consists of a complete bipartite graph with an additional vertex which has exactly two neighbours, and these are on the same side of the bipartition. As special cases, this proves Scott's conjecture when H is obtained from K_4 by subdividing two disjoint edges.

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Matching Problems in Temporal Graphs

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Abstract

We study temporal graphs, whose topology changes discretely over time as edges can disappear and then reappear. This flexibility makes them a natural model for many optimization problems, including maximum matching, which itself can be formulated in several ways in this setting. We consider three extensively studied matching models: Δ-matchings, γ-matchings, and d-distance matchings. We focus on the fundamental setting in which the underlying graph is a tree, and provide a complete picture of computational complexity, including hardness results, polynomial-time solvable cases, approximation algorithms, and structural connections between the models. Our main results are as follows. We prove that the maximum Δ-matching problem is NP-hard on trees for every $\Delta \geq 2$, even when each edge appears at most twice. We also show that the maximum γ-matching problem is NP-hard on trees as well, even if there are at most two continuous intervals with length γ in $\lambda(e)$ for every edge e. Complementing these hardness results, we identify natural polynomial-time solvable cases. For instance, the maximum Δ-matching can be solved in polynomial time if every edge appears at most once. We also prove that the d-distance matching problem is polynomial-time solvable when the input bipartite graph is a tree. Finally, we design efficient polynomial-time approximation schemes (EPTASs) for the maximum Δ-matching and maximum γ-matching on trees. Our proofs use polynomial reductions that preserve tree structure, the relationship between totally unimodular matrices and flows, and dynamic programming over time-window decompositions that exploit temporal locality.

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A quasi-polynomial bound for the minimal excluded minors for a surface

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Abstract

As part of their graph minor project, Robertson and Seymour showed in 1990 that the class of graphs that can be embedded in a given surface can be characterized by a finite set of minimal excluded minors. However, their proof, because existential, does not provide any information on these excluded minors. Seymour proved in 1993 the first and, until now, only known upper bound on the order of the minimal excluded minors for a given surface. This bound is double exponential in the Euler genus g of the surface and, therefore, very far from the $\Omega(g)$ lower bound on the maximal order of minimal excluded minors for a surface and most likely far from the best possible bound. More than thirty years later, this paper finally makes progress in lowering this bound to a quasi-polynomial in the Euler genus of the surface.

The main catalyzer to reach a quasi-polynomial bound is a breakthrough on the characteristic size of a forbidden structure for a minimal excluded minor G for a surface of Euler genus g: although it is not hard to show that G does not contain $O(g)$ disjoint cycles that are contractible and nested in some embedding of G as demonstrated by Seymour, this bound can be lowered to $O(\log g)$ which is essential to obtain the quasi-polynomial bound in this paper.

As subsidiary results, we also improve the current bound on the treewidth of a minimal excluded minor G for a surface by improving the first and, until now, only known bound provided by Seymour. Moreover, we show a better upper bound on the order of a grid minor in G, improving a result by Thomassen from 1997.

This result was presented at the 2026 SODA conference.

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Accumulation Points of Homomorphism Thresholds

Xinqi Huang*

Abstract

For a fixed graph H, the *chromatic threshold* $\delta_\chi(H)$ is the infimum of all $\alpha \in [0,1]$ such that every sufficiently large H-free graph G with $\delta(G) \geq \alpha|V(G)|$ has bounded chromatic number. This notion goes back to Erdős and Simonovits in 1973, and was completely determined for all fixed graphs by Allen, Böttcher, Griffiths, Kohayakawa, and Morris in 2013: for each integer $r \geq 3$, the chromatic thresholds of r-chromatic graphs take only three possible values.

Motivated by Thomassen's 2002 theorem on dense triangle-free graphs, one is led to the stronger notion of the *homomorphism threshold*, which asks for a bounded homomorphic image that is itself H-free. In striking contrast with the chromatic-threshold theory, homomorphism thresholds remain largely mysterious: for many years, the clique case was essentially the only family whose exact homomorphism threshold was known. A recent breakthrough of Sankar introduced topological methods to show that every odd cycle has positive homomorphism threshold, yielding the first broad separation between chromatic thresholds and homomorphism thresholds.

In this talk, we show that homomorphism thresholds exhibit a dramatically richer behaviour than chromatic thresholds. For every integer $r \geq 3$, we obtain infinitely many new values of homomorphism thresholds arising from graph classes of chromatic number r, and we further identify an accumulation point for each such family of values. Consequently, the set of homomorphism thresholds has infinitely many accumulation points. As a corollary, our results demonstrate from several directions that chromatic thresholds and homomorphism thresholds behave fundamentally differently.

This talk is based on joint work with Lior Gishboliner and Hong Liu.

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Three Cops Win on the Torus (even if only two can move at a time)

Florian Lehner*

Abstract

Cops and Robber is a pursuit-evasion game played on a graph between two players which can be described as follows. Initially, the first player, places k cops on vertices of a graph G, then the second player places a robber on a vertex. Then the two players take turns. On the first player's turn each cop can either be moved to an adjacent vertex or left at the current position, on the second player's turn the robber can either be moved to an adjacent vertex or left where he is. The first player wins the game if at some point one of the cops is at the same vertex as the robber. The l-cops-move game is defined analogously, with the only difference that at most l of the cops can move on each cop turn.

A classical result by by Aigner and Fromme states that every planar graph admits a winning strategy for 3 cops if all of them are allowed to move in every cop turn. While no upper bound for the number of cops needed to guarantee a win in the 1-cop-moves game on planar graphs is known, González Hermosillo de la Maza and Mohar recently showed that 3 cops still have a winning strategy in the 2-cops moves game. We show that an analogous result holds for graphs embeddable in the torus, projective plane, or Klein bottle.

This talk is based on joint work with Alan Sun.

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